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BASIS:
A Data Structure for Adaptive Multigrid Computations

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Abstract

In this report a data structure and basic algorithms are described, that can be used for
the implementation of adaptive multigrid procedures. The basic elements of the grid are
rectangular cells that fit in a QUAD-tree structure. A PASCAL prototype implementation
is given and also a FORTRAN implementation is available.

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1 The geometric structure

1.1 Introduction

In this report we describe a data structure that can be used for the solution of partial differ-
etial equations. It is meant for the implementation of multigrid methods and handling the
corresponding data on a self-adaptive mesh. In particular, it will be applied in a program
for the solution of the Euler- and compressible Navier-Stokes equations. A generalization to
three or more dimensions is straightforward, only for notational convenience it is not explicitly
described here.

We assume that the domain of definition of the partial differential equations, \( \Omega \in \mathbb{R}^2 \),
satisfies sufficient conditions, in order that -possibly after a suitable transformation- a regular
partitioning by quadrangles can be made. The boundary of \( \Omega \) is denoted by \( \partial \Omega \).

1.2 Cells and neighbours

The domain of definition \( \Omega \) is divided into a regular partitioning of a finite number of quad-
rangles \( \Omega_{i,j}^0 \),

\[
\Omega = \bigcup_{(i,j) \in I} \Omega_{i,j}^0,
\]

(1)

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in such a way that all quadrangles have, in each of the directions North, East, South or West, 
either one or two neighbour quadrangle $\Omega_{i,j,1}^1$, $\Omega_{i,j,1}^0$, $\Omega_{i,j,-1}^0$ or $\Omega_{i,j,-1}^1$. This means that for each 
$(i,m) = (0,1), (1,0), (0,-1), (-1,0)$ the cell $\Omega_{i,j}^0$ shares either no or exactly one complete 
edge with the quadrangle $\Omega_{i,j}^0$. The index set $I$ is a finite subset from either $\mathbb{Z}^2$ or 
$\mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2}$, for some natural numbers $N_1$ or $N_2$, so that the domain $\Omega$ may be (part of) a 
cylinder or torus. In this way, for each cell its N, E, S or W- neighbour cell (if it exists) is 
well determined. For each cell we further can identify its N, E, S and W edge and its NE, SE, 
SW and NW corner. However, we notice that an edge can have positive or zero length. This 
allows the possibility of cells degenerating to triangles, or even cells of which the area vanishes.

The set $L^0 = \{ \Omega_{i,j}^0 | \Omega_{i,j}^0 \subset \Omega \}$ is the set of cells on level zero. It exactly covers the domain $\Omega$.

Remark 1.1 The structure of $\Omega$ and its actual subdivision is not changed if the cells are 
renumbered in such a way that a constant integer vector $(c_1, c_2)$ is added to the index vector 
$(i,j)$. Therefore, without loss of generality we can number the cells so that the lowest indices 
are zero, so that $\min\{i| \Omega_{i,j}^0 \subset \Omega \} = 0$, and $\min\{j| \Omega_{i,j}^0 \subset \Omega \} = 0$.

A cell $\Omega_{i,j}^k$ on level $k$ may be divided into 4 disjoint cells on level $k+1$,

$$
\Omega_{i,j}^k = \bigcup_{i,m=0,1} \Omega_{i,j}^{k+1,2i+2j+m}.
$$

Remark 1.2 Cells or levels $k > 0$ always appear in quadruples.

Remark 1.3 The division in cells on finer levels (i.e. levels with a larger $k$) are always made 
in such a way that two cells share either no edge at all, or they share a complete edge.

Remark 1.4 The division is made such that in the limit for $k \to \infty$, the length of the largest 
edge on level $k$ vanishes, i.e. $\lim_{k \to \infty} h_k = 0$.

Definition 1.5 (level $k$) The set $L^k = \{ \Omega_{i,j}^k | \Omega_{i,j}^k \subset \Omega \}$ is the set of cells on level $k$. The 
number of cells in $L$ is denoted by $N_k$. (For $k > 0$, $N_k$ satisfies $N_k \leq 4N_{k-1}$.)

Definition 1.6 ((\xi, \eta)-coordinates) We provide $\Omega$ with a $\xi$, $\eta$)-coordinate system such that 
for the South-West corner of each cell $\Omega_{i,j}^k$ the coordinates are given by

$$
(\xi, \eta) = (2^{-k}, j \cdot 2^{-k}).
$$

Remark 1.7 It is easy to see that the construction of such a coordinate system is possible and 
that each point in $\Omega$ is determined by a pair of coordinates. This coordinate system has little 
relation with the coordinate system(s) in possible applications. The present $(\xi, \eta)$ coordinate 
system determines the topological structure of the domain.

Definition 1.8 (neighbours) Two cells $\Omega_{i,j}^k$ and $\Omega_{i,m}^n$ are called neighbours if $k = n$ and 
$|i-l| + |j-m| = 1$.

Remark 1.9 Each cell can have 0,1,2,3 or 4 neighbours.

Definition 1.10 (wall) An edge of a cell is also called wall.

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1In this report the orientation of the winds, as well as the meaning of the words 'left', 'right', 'bottom' and 'top' are related in the usual way with the $(i,j)$-indices.
1.3 Ghost cells, parents and kids

Remark 1.11 Because two cells either share no edge at all or a complete edge, each edge has either 1 or 2 neighbouring cells.

Definition 1.12 (V-type, H-type) A wall between two cells $\Omega_{i,j}^k$ and $\Omega_{i+1,j}^k$ is called of V-type; a wall between two cells $\Omega_{i,j}^k$ and $\Omega_{i,j+1}^k$ is called of H-type.

Definition 1.13 (corner points) End points of walls are corner points.

By the regular partitioning in cells, at each corner point at most 4 cells meet. This is easily seen by means of the $(i, \eta)$-coordinate system.

1.3 Ghost cells, parents and kids

In order to form a QUAD tree of cells with a single root, we have to combine cells on level 0 in groups on level -1, and recursively, groups on level k in groups on level k-1, until they are all combined in a single group that encloses them all. Therefore we introduce 'ghost cells' (i.e. groups of cells) in the following way. Notice that ghost cells are not cells!

Definition 1.14 (ghost cell) A ghost cell $\Omega_{i,j}^{-k}$, $k > 0$, is

$$\Omega_{i,j}^{-k} = \bigcup_{l,m=0,1} \Omega_{2i+4,2j+m}^{-k+1}.$$  \hspace{1cm} (4)

Definition 1.15 (enclosing level) The smallest $-k$ for which only one ghost cell (or cell) exists is called the enclosing level, $k_1$.

Remark 1.16 The enclosing level $k_1$ satisfies $k_1 \leq 0$.

Definition 1.17 (enclosing cell) The (ghost) cell on the enclosing level is called the enclosing cell.

Remark 1.18 By the choice of the indices $(i,j)$ on the level $k = 0$, such that the left-most boundary fits with $i = 0$ and the bottom with $j = 0$, it is clear that the enclosing cell is identified by $\Omega_{0,0}^0 = \Omega$.

Definition 1.19 (parent, kid cell) For a family of (ghost) cells on the level $k$ and $k+1$, related by

$$\Omega_{i,j}^{k} = \bigcup_{l,m=0,1} \Omega_{2i+4,2j+m}^{k+1}, \hspace{1cm} k \geq k_1,$$  \hspace{1cm} (5)

the (ghost) cell $\Omega_{i,j}^{k}$ is called the parent and $\Omega_{2i+4,2j+m}^{k+1}$ are called the kids.

Definition 1.20 (internal, boundary, green wall) A wall that has two neighbouring cells is called an internal wall. If it has only one neighbour cell, it is called a boundary wall if it is part of $\partial \Omega$, or it is called a green wall if it is not part of $\partial \Omega$.

Definition 1.21 (internal, boundary, green point) If a corner point is the endpoint of a boundary wall, it is called a boundary point. If it is the endpoint of a green wall, it is called a green point. If it isn't a boundary point or green point, it is called an internal point.

Remark 1.22 A corner point can be both a boundary and a green point at the same time. Notice that 4 cells and 4 walls meet at a corner point that is an internal point,
Definition 1.23 (internal, boundary, green cell) If a cell has a boundary wall, and hence a boundary point as a corner point, it is called a boundary cell. Similarly a cell is a green cell if it has a green wall. If a cell is not a boundary or green cell, it is an internal cell.

Summary
- There is one enclosing cell that exactly covers \( \Omega \);
- A ghost cell lives on negative levels and can have 1,2,3 or 4 kids;
- A cell lives on non-negative levels and can only have 0 or 4 kids;
- We don’t associate walls and corner points with ghost cells. (No walls or corner points are defined for negative levels);
- A cell has 4 walls (N,E,S or W), and 4 corner points (NE, SE, SW or NW) that possibly coincide;
- A wall has 1 or 2 neighbour cells;
- A wall is either of H-type or of V-type;
- A corner point can have 1,2,3 or 4 neighbour cells (in any combination, i.e. 15 possible relative locations);
- On level \( k = 0 \) a cell can have 0,1,2,3 or 4 neighbours; on level \( k > 0 \) a cell has 2,3 or 4 neighbours;
- A corner point can have 2,3 or 4 neighbour corner points;
- Two cells meet at an interior wall;
- Non-interior walls are either boundary or green walls;
- Four cells meet at an interior point;
- Non-interior points can be boundary and/or green points;
- Green points and walls can only exist for levels \( k > 0 \).

1.4 The geometric system

Here we give a numbering system for walls and corner points. Of course, walls and corner points are identified by their level and their \((\xi, \eta)\)-coordinates. For an easier reference, and in order to introduce a numbering system that can be used in a computer implementation, we assume the following conventions:

- \( P^k_{i,j} \) is the corner point on level \( k \), with coordinates \( P^k_{i,j} = (i 2^{-k}, j 2^{-k}) \);
- \( \Gamma^k_{V,i,j} \) is the wall on level \( k \), with coordinates \( \{(i 2^{-k}, t 2^{-k}) | j \leq t \leq j + 1 \} \);
- \( \Gamma^k_{H,i,j} \) is the wall on level \( k \), with coordinates \( \{(t 2^{-k}, j 2^{-k}) | i \leq t \leq i + 1 \} \).
- All items \( P^k_{i,j}, \Gamma^k_{V,i,j}, \Gamma^k_{H,i,j} \) and \( \Omega^k_{i,j} \) that exist, together form the geometric system.

By the selection of the SW-corner in the definition 1.6 we have introduced an asymmetry in the data structure, in the sense that the point \((i 2^{-k}, j 2^{-k})\) is not the centre of the cell \( \Omega^k_{i,j} \). This asymmetry could easily be removed by introducing half-indices \( P^k_{i-1/2,j-1/2} \) or \( \Omega^k_{i+1/2,j+1/2} \). However, because it is our intention to end up with an actual implementation for which indices better can be successive integers, and data in an array can better be of the same type, we accept the minor inconvenience of the asymmetry.
1.5 The patch

Definition 1.24 (patch) To each corner point $P_{i,j}^k$ in the geometric system we associate a patch of data

$$\Pi_{i,j}^k = (P_{i,j}^k, \Gamma_{V_{i,j}}, \Gamma_{R_{i,j}}, \Omega_{i,j}^k), \quad k \geq 0,$$

where possibly $\Gamma_{V_{i,j}}$, $\Gamma_{R_{i,j}}$ and $\Omega_{i,j}^k$ are empty if they do not exist in the geometric system.

Remark 1.25 Because each cell has a single SW corner point and each wall has either a single S or W end point, all cells and all walls can be found in exactly one patch.

Remark 1.26 Because each cell has 4 corner points, the complete description of a cell - including its walls and corner points - always needs 4 patches.

Definition 1.27 (parent patch) With any non-empty set of patches \{\Pi_{2i+1,2j+m}^{k+1}; l, m = 0, 1\} we associate its parent patch $\Pi_{i,j}^k$.

Definition 1.28 (kid patch) Patches that share the same parent $\Pi_{i,j}^k$ are called the kid-patches of this parent.

Remark 1.29 By the definitions it follows immediately that if a parent-kid relation exists between two (ghost) cells $\Omega_{i,j}^k$ and $\Omega_{2i+1,2j+m}^{k+1}$, then the same relation exists for the patches $\Pi_{i,j}^k$ and $\Pi_{2i+1,2j+m}^{k+1}$.

Definition 1.30 (root level) The smallest $k$ for which only one patch exists is called the root level, $k_0$.

Definition 1.31 (root patch) The patch on the root level is called the root patch.

Remark 1.32 The root level is $\Pi_{0,0}^{k_0}$; the root level $k_0$ satisfies $k_0 = k_1 - 1 < 0$.

Definition 1.33 (ghost patch) The patches $\Pi_{i,j}^k$ with $k_0 \leq k < 0$ are called ghost patches

$$\Pi_{i,j}^k = (\emptyset, \emptyset, \emptyset, \Omega_{i,j}^k), \quad k < 0,$$

where possibly $\Omega_{i,j}^k$ is empty.

Remark 1.34 Ghost patches are patches!
Remark 1.35 Notice that there are more ghost patches than ghost cells.

Definition 1.36 (complete patch) If \( k \geq 0 \) and \( \Omega_{i,j}^k \) is not empty, then also \( \Gamma_{V,i,j}^k \) and \( \Gamma_{H,i,j}^k \) are not empty, and the patch is called complete.

Definition 1.37 (H- or V-patch) If \( k \geq 0 \) and \( \Gamma_{H,i,j}^k \) (or \( \Gamma_{V,i,j}^k \)) is not empty, then the patch is called a H- or V-patch, respectively.

Definition 1.38 (thin patch) If \( k \geq 0 \) and a patch is not complete, it is called thin. This patches can be called V-thin (if \( \Gamma_{V,i,j}^k \) exists), and H-thin (if \( \Gamma_{H,i,j}^k \) exists).

Remark 1.39 Thin patches can be any combination of V- and H-thin; they can be \( V \), \( H \), \( IV \), and \( 0 \)-thin. The patch is \( 0 \)-thin if only its corner-point contributes to the geometric system. It is \( HV \)-thin in the case of a reentrant corner.

Remark 1.40 By construction, it is clear that for all positive levels, a cell or patch has all kids available if and only if its NE-kid exists.

Definition 1.41 (neighbour patch) For each patch \( \Pi_{i,j}^k \), we define its neighbours \( \Pi_{i,m}^k \) by the requirement \( |i-l| \leq 1 \), \( |j-m| \leq 1 \). Similar to the cells we can identify patches as N-, E-, S- or W-neighbours.

Summary

- There exists exactly one root patch on the root level \( k_0 \);
- There exist ghost patches \((k_0 \leq k < 0)\); possibly they contain a ghost cell; they contain no walls or corner points; they have neighbour and parent-kid relations;
- There exist patches that are complete \((k \geq 0)\); they contain a cell, two walls and a corner point;
- There exist patches that are not complete \((k \geq 0)\); they contain a corner point, possibly one or two walls and no cell;
- Patches that are complete have \((k \geq 0)\): 1 parent; 0,1,2,3 or 4 kids; 0,1,2,3 or 4 neighbours; complete data contents;
- Patches that are thin (not complete) have \((k \geq 0)\): 1 parent; 0,2 or 3 kids; 1,2,3 or 4 neighbours; partial data contents.

1.6 Conclusion

The patches, introduced in section 1.5, together with the parent-kid relations form a tree. The root patch is found on level \( k_0 \). The patches on the levels \( k \geq 0 \) are all associated with corner points in the geometric structure.

The subset of patches \( \Pi_{i,j}^k \), for which there exists a (ghost) cell \( \Omega_{i,j}^k \) in the geometric structure, forms a genuine sub-tree in the tree of patches. The patches in this sub-tree that are on levels \( k \geq 0 \), are the complete patches, which contain the cells in the geometric structure.
2 The data structure

The data structure is ordered by patches. Hence it forms a tree with at most 4 new branches at each node: the QUAD tree. This is the basic structure in which all data about the problem are stored. In the following (sub)sections we describe how the tree is implemented and how operations on it can be executed. One of the aims of the data structure is to provide the means to program multigrid algorithms for adaptive mesh refinement. These algorithms can be based on cell centered schemes as well as based on vertex centered schemes.

With each patch we also associate some pointers, properties, coordinates and data contents, of which different parts are associated with the corner point, V-wall, H-wall and/or the cell in the patch. The data contents is any kind of information in the form of real or integer numbers etc. related with the elements of the geometric system. Each patch, except the root patch, contains a pointer to its parent, and all patches contain pointers to their existing kids. Further, each patch has pointers to all its existing neighbour patches. To indicate the non-existence of kids or neighbours a nil pointer is used.

2.1 The implementation

Although PASCAL is really well suited to properly implement the data structure that is described in this report, for some trivial but practical reasons it can be decided that it should be implemented (also) in FORTRAN.

The experience is that the construction of a FORTRAN implementation is enhanced if first a prototype is made available in a better equipped language. Therefore, a prototype of the essential parts of the data structure is built in PASCAL, taking into account the essential restrictions that are inherent to the use of FORTRAN. The useful features as pointers and recursive procedures, that are available in PASCAL but not in FORTRAN, were abandoned in this prototype.

Because the data structure is organized by patch, each patch is given its unique natural number. The number of the root patch is 1. The other patches are sequentially ordered, in order of appearance. Only to accommodate the implementation, we have to specify the maximum number of patches 'MaxNumberOfPatches'.

The pointers in the patch are implemented as integers referring to the corresponding (parent-, neighbour- or kid-) patch. Properties of patches are implemented in a Boolean (logical) array. The coordinates are given in an integer array. The data contents is (mainly) found in the real or double precision arrays of the data structure.

2.2 Pointers

An integer array 'PNTR', dimensioned (FirstPointer: LastPointer: 0: MaxNumberOfPatches), is used for keeping the pointers. For each patch a set of nine pointers is reserved.

<table>
<thead>
<tr>
<th>Array element</th>
<th>the integer referring to</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNTR(Parent, Patch)</td>
<td>the parent of Patch;</td>
</tr>
<tr>
<td>PNTR(NE, Patch)</td>
<td>the NE kid of Patch;</td>
</tr>
<tr>
<td>PNTR(SE, Patch)</td>
<td>the SE kid of Patch;</td>
</tr>
<tr>
<td>PNTR(SW, Patch)</td>
<td>the SW kid of Patch;</td>
</tr>
<tr>
<td>PNTR(NW, Patch)</td>
<td>the NW kid of Patch;</td>
</tr>
<tr>
<td>PNTR(N, Patch)</td>
<td>the N neighbour of Patch;</td>
</tr>
<tr>
<td>PNTR(E, Patch)</td>
<td>the E neighbour of Patch;</td>
</tr>
<tr>
<td>PNTR(S, Patch)</td>
<td>the S neighbour of Patch;</td>
</tr>
<tr>
<td>PNTR(W, Patch)</td>
<td>the W neighbour of Patch;</td>
</tr>
</tbody>
</table>

The nil pointer is the pointer that refers to a nonexisting patch. The nil pointer is implemented
as 0 (zero).

**Remark 2.1** We dimension the array PNTR as \((\text{FirstPointer} : \text{LastPointer}, 0 : \text{MaxNumberOfPatches})\). For all the patches the information about the pointers is contained in PNTR \((*,1: \text{MaxNumberOfPatches})\). The elements of PNTR\((*,0)\), the pointers of the nil patch, are all initialized (and kept) equal to zero. That makes that any pointer from the nilpatch is again the nilpointer. This simplifies the implementation because it makes that "the kid or neighbour of a not-existing patch doesn't exist".

**Remark 2.2** Whereas the order of the pointers has no intrinsic meaning, the actual implementation is made by named integer constants\(^2\). However, we fix the order of the winds so that the ordering of these eight winds can be used in a loop\(^3\).

The index bounds are given by FirstPointer\(\leq 0\) and LastPointer\(\geq 0\). These values can be changed in order to accommodate the storage of the integer data if necessary. In fact, in the FORTRAN implementation we will have FirstPointer\(\leq 3\), because the integer array Location\((0:2,*)\), as described in section 2.4, is actually stored as Location\((i,j) = \text{PNTR}(i-3,j)\) (cf. section 3.4).

### 2.3 Properties

Various properties of a patch will be available in a Boolean array. Many of these properties (e.g., whether the patch is located near a boundary) can be derived from the pointer structure, but to avoid many trivial recomputations, some properties can better be stored independently in the data structure. (A routine CHKPT can be available to check the consistency of the data, to report and, eventually, to correct the properties.) Some other (additional) information in the form of properties, can be defined by the user.

Properties of a patch that are of interest are the following: complete, thin (HV, H, V or 0). Properties of the corresponding (ghost) cell are: ghost cell or genuine cell; boundary cell, green cell or internal cell; all (NE, SE, SW and NW) kids available. Properties of the H- or V-wall are: boundary wall, internal wall, green wall. Information that is provided by the user can e.g. be the type of boundary condition (Dirichlet, Neumann, etc.).

Notice that for all positive levels, a cell or patch has all kids available if and only if its NE-kid exists (cf. 1.40). Therefore no properties are introduced to denote the existence of kids for a particular cell.

Other properties of interest for the data structure are 'pregnant', 'sentenced', and 'dead'. The property 'pregnant', for a complete patch, is set to true if the corresponding cell is to be refined at the next occasion. Similarly 'sentenced' is used to denote that the cell is to be deleted at the next occasion. A 'dead' patch is the space that remains when a patch has been removed from the data structure. This space contains only garbage to which no pointer is referring. At the next creation of a new patch, this space can be reused.

Some of the above property data are stored in the Boolean (logical) array PPTY, dimensioned \((\text{FirstPpty}: \text{LastPpty}, 1: \text{MaxNumberOfPatches})\)

<table>
<thead>
<tr>
<th>Array element(^4)</th>
<th>referring to</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPTY(Complete, Patch)</td>
<td>Complete patch;</td>
</tr>
<tr>
<td>PPTY(WallH, Patch)</td>
<td>(I_{h,i,j} \in \Pi_{i,j})</td>
</tr>
<tr>
<td>PPTY(WallV, Patch)</td>
<td>(I_{v,i,j} \in \Pi_{i,j})</td>
</tr>
</tbody>
</table>

\(^2\) FirstPointer\(\leq 0\), LastPointer\(\geq 8\), Parent=0, NE=1, SE=2, SW=3, NW=4, N=5, E=6, S=7, W=8

\(^3\) Although the (main) purpose of the use of named integer constants is that the number of pointers etc. can be extended and that the order can be shuffled, under no circumstances the numbering of the winds will be changed!
2.4 Co-ordinates

\text{PPTY(BdyPoint, Patch)} \quad P_{i,j} \in \partial \Omega
\text{PPTY(BdyWallH, Patch)} \quad \Gamma_{H,i,j} \in \partial \Omega
\text{PPTY(BdyWallV, Patch)} \quad \Gamma_{V,i,j} \in \partial \Omega
\text{PPTY(BdyCell, Patch)} \quad \Omega_{i,j} \text{ touches the boundary;}
\text{PPTY(GrnPoint, Patch)} \quad P_{i,j} \text{ is endpoint of a green wall;}
\text{PPTY(GrnWallH, Patch)} \quad \Gamma_{H,i,j} \text{ is a green wall;}
\text{PPTY(GrnWallV, Patch)} \quad \Gamma_{V,i,j} \text{ is a green wall;}
\text{PPTY(GrnCell, Patch)} \quad \Omega_{i,j} \text{ touches a green wall;}
\text{PPTY(Pregnant, Patch)} \quad \text{to be refined at next occasion;}
\text{PPTY(Sentenced, Patch)} \quad \text{to be deleted at next occasion;}
\text{PPTY(Dead, Patch)} \quad \text{a garbage patch, to be reused;}

\textbf{Remark 2.3} Where the order of the Boolean in the array has no intrinsic meaning, the actual implementation is made by named integer constants\(^4\).

\textbf{Remark 2.4} Another possibility could be to implement the properties in a character array (FirstPty:LastPty:1:MaxNumberOfPatches), or in a character string array: character*14 (1:MaxNumberOfPatches). The advantages and disadvantages are not yet completely clear (convenience, run-time speed, storage requirements). An advantage might be that a selection from more than two possibilities can be made. E.g. by a character array one could mark a cell as complete, thin or ghost; whereas by a Boolean we only distinguish between complete or not complete.

\textbf{Remark 2.5} The local situation, e.g. the location with respect to a neighbouring boundary, can directly be derived from the properties stored in PPTY and in PNTR. To check the consistency of these data with the actual pointer data the routine CHKPPT can be available.

2.4 Co-ordinates

The geometric data of the structure, in the form of \((\xi, \eta)\)-coordinates, are stored in an integer array Location (0:2, 1:MaxNumberOfPatches), where Location(0, Patch) gives the level \(k\) of the patch \(L^k\), and Location(1, Patch) and Location(2, Patch) yield the integers \(i\) and \(j\) respectively.

Generally, the real (physical or \((x,y)\)-) coordinate system will be quite different from the \((\xi, \eta)\)-coordinate system. To have the real coordinates available, we can proceed in three ways. (1) The real coordinates are made available in the form of a routine that implements the mapping \(\xi = (x, y);\) (2) a real data array is made available COORD (1:2, 1:MaxNumberOfPatches) in which the real coordinates can be found for each point in the structure; or (3) the real coordinates are found in an unformatted direct accessible file (if these data are not frequently accessed).

2.5 Data Contents

The data contents of the data structure is contained in the REAL or DOUBLE PRECISION arrays dimensioned as

\text{DCELL} \quad (1:LastCellData, 1:MaxNumberOfPatches)
\text{DH WALL} \quad (1:LastWallData, 1:MaxNumberOfPatches)

\(^4\text{FirstPty} \leq 1, \text{LastPty} \geq 14, \text{Complete} = 1, \text{WallH} = 2, \text{WallV} = 3, \text{BdyPoint} = 4, \text{BdyWallH} = 5, \text{BdyWallV} = 6, \text{BdyCell} = 7, \text{GrnPoint} = 8, \text{GrnWallH} = 9, \text{GrnWallV} = 10, \text{GrnCell} = 11, \text{Pregnant} = 12, \text{Sentenced} = 13, \text{Dead} = 14.$
where e.g. DHWALL(Length, Patch) gives the length of the horizontal wall in the patch 'Patch'; 'length' being one of the WallData.

Possible 'PointData' are e.g. the function values of the unknown (cell vertex algorithms). Possible 'WallData' are e.g. the length, sine- and cosine of the normal vector on the wall. Possible 'CellData' are e.g. the function values of the unknown (cell centered algorithms) and/or the residual.

Remark 2.6 The user can add his own additional arrays as additional storage for data in the structure, provided that it is dimensioned '1:MaxNumberOfPatches'.

2.6 Construction of the data structure

For the construction and the handling of the data structure, the following routines should be available to the user. Notice that routines are provided both for creating parts of the data structure, as well as removing parts of it. The space that is made free by the removal of some parts will be used by the newly generated parts.

- **routine GetRectangle(n,m).** Creates the data structure for the domain \( \Omega \), where \( \Omega \) is (topologically equivalent with) a rectangle. At level 0, \( \Omega \) is a rectangle with \( m \times n \) cells.

- **routine GetCylinder(n,m).** Is similar to GetRectangle. However, the domain \( \Omega \) at level 0 is a cylinder with \( m \times n \) cells. The cylinder is located such that the N-side of a rectangle is identified with the S-side. That means that the cylinder has V-walls but no H-walls.

- **routine Offspring(Ptr).** Creates the 4 kids for a complete patch, that is identified by Ptr, and that has no kids yet. It makes an update of the data structure, so that it is consistent with the new status.

- **routine Educate(Ptr).** Fills the data contents of the kids of Ptr, as can be derived from its own contents. This procedure is dependent on the application and should be provided by the user.

- **routine RemoveOffspring(Ptr).** Removes the four kids of the cell identified by Ptr. This routine can be used to remove refinements that are not longer necessary.

- **routine KillCell(Ptr).** Removes the cell identified by Ptr. This routine can be used to re-shape a rectangle or cylinder on level 0 into an arbitrarily shaped region.

Some of the routines (mentioned below) are only auxiliary for the other routines and are not of immediate use to the user.

- **integer function NewKid.** Creates a new patch as one of the four kids of a parent patch. Auxiliary routine.

- **integer function MakePatch.** Creates a new patch and delivers its pointer. Auxiliary routine.

- **routine RemovePatch(Ptr).** If possible (i.e. if neighbour cells don't need this patch for its corner point), this routine removes the patch identified by Ptr from the data structure. Auxiliary routine. (After a successful removal of the patch it is labelled as reusable space by the property Dead.)

---

\(^{5}\)Length is a constant, \( 1 \leq \text{Length} \leq \text{LastWallData} \).
3 The actions on the data structure

3.1 Scanning patches

Because the implementation in FORTRAN prevents the recursive treatment of essentially recursive algorithms, we provide here the non-recursive versions, in order to document the necessary logical spaghetti.

In the following pseudo-code we describe how by a non-recursive algorithm the QUAD tree is searched. This search can be made e.g. for all or part of the patches or cells in the tree. Therefore, besides the root pointer of the (sub-) tree to be scanned (RootPointer), we also specify the lowest (FromLevel, $k_4$) and highest (ToLevel, $k_1$) level of the patches $\{\Pi_{p, j}^k | k_4 \leq k \leq k_1\}$ to be visited. The order in which the patches are scanned is given by the parameter 'Order'. This is an integer array Order(1:4), which contains a permutation of the directions NE, SE, SW, NW.

```
routine Scan (RootPointer, Order, FromLevel, ToLevel, Doit );
begin
if RootPointer \neq NilPointer then
    LowLevel := LevelOf(RootPointer)
    IPTR(LowLevel) := RootPointer
    if LowLevel = FromLevel then DoIt(RootPointer) endif

if ToLevel > LowLevel then
    LowLevel + := 1
    for lev from LowLevel to ToLevel do i(lev) := 0 enddo
    lev := LowLevel
repeat
    i(lev) + := 1
    if i(lev) \leq 4 then
        IPTR(lev) := PNTR(Order(i(lev)),IPTR(lev-1))
        if IPTR(lev) \neq NilPointer then
            if lev < ToLevel then
                if lev \geq FromLevel then DoIt(IPTR(lev)) endif
                lev + := 1
            else
                DoIt(IPTR(lev))
            endif
        else if lev > LowLevel then
            i(lev) := 0
            lev - := 1
        endif
    until i(LowLevel) > 4
end
end
```

In fact, the routine SCAN scans all patches. The routine scans the existing patches in the specified order, and visiting each patch, it makes a call to the procedure DoIt(PointerToPatch). The actual routine for 'DoIt' can operate on the patch.
3. THE ACTIONS ON THE DATA STRUCTURE

We might need the following routines, that scan patches (cells, walls, H-type walls, V-type walls or points). However, by means of the properties given in PPTY, the implementation of these routines is trivial.

\begin{verbatim}
ScanPatches (RootPointer, Order, FromLevel, ToLevel, DoIt)
ScanCells  (RootPointer, Order, FromLevel, ToLevel, DoIt)
ScanWalls  (RootPointer, Order, FromLevel, ToLevel, DoIt)
ScanHWalls (RootPointer, Order, FromLevel, ToLevel, DoIt)
ScanVWalls (RootPointer, Order, FromLevel, ToLevel, DoIt)
ScanPoints (RootPointer, Order, FromLevel, ToLevel, DoIt)
\end{verbatim}

3.2 The FMG routine

Here we describe in pseudo-code the structure of the global multigrid routine FMG. The integer array nfas(bottomlevel:maxlevel) contains part of the multigrid strategy.

\begin{verbatim}
routine Fmg (maxlevel,nfas,npmg,nqmg,ncycl,f)
begin
  for toplevel from bottomlevel to maxlevel do
    for i from 1 to nfas(i) do Fas(toplevel) enddo
    \( q^H_{\text{toplevel}+1} := F_{\text{toplevel}}^H \)
  enddo
end
\end{verbatim}

3.3 The FAS cycle

In this section we describe in non-recursive pseudo-code the structure of the nonlinear multigrid routine FAS.

Given are the strategy parameter arrays npmg (bottomlevel:toplevel), ncycl (bottomlevel:toplevel) and nqmg (bottomlevel:toplevel). For each level, from bottomlevel to toplevel, these arrays determine the number of pre- or post-relaxation cycles in each FAS-cycle (in npmg and nqmg respectively). In array ncycl the number of coarse grid correction cycles is given. These are positive integers (non-zero); ncycl(i)=1 gives a V-cycle, ncycl(i)=2 gives a W-cycle. Also an additional integer \( f \) denotes the use of an F-cycle: if \( f = 1 \) an F-cycle will be made, otherwise \( f = 0 \). Notice that V-F-cycles and W-F-cycles are possible!

\begin{verbatim}
routine Fas (bottomlevel, toplevel, npmg, nqmg, ncycl, f)
begin
  for i from bottomlevel to toplevel do iters(i):= f enddo
  iters(toplevel) := 1
  level := toplevel

  while iters (toplevel) > 0 do
    while level > bottomlevel do
      PRE-RELAXATION
      for i from 1 to npmg(level) do relax(level) enddo
      RESTRICTION
      \( r_H := R_{HH}(r_k - N_{HqH}) \)
      \( q_H := r_H + N_{HqH} \)
      \( q^0_H := q_H \)
  enddo
enddo
end
\end{verbatim}

\* These are probably less urgent, because, on a positive level, points and patches are similar for this purpose.
3.4 The FORTRAN implementation

level := 1
iters(level) := ncycl(level)
enddo

SOLVE ON COARSEST GRID
(Here always level = bottomlevel)
for i from 1 to iters(level) do solve (level) enddo
iters(level) := 0

while (level < toplevel) and (iters(level) = 0) do
PROLONGATION
$q_h := q_h + P_h H (q_H - q_H^0)
level := level + 1
POST-RELAXATION
for i from 1 to nqmg(level) do relax(level) enddo
iters(level) := 1
enddo
enddo

3.4 The FORTRAN implementation

In the Fortran implementation all data about the data structure are collected in three ARRAYS and a COMMON BLOCK. The arrays that are declared in the main program are: the integer array PNTR, dimensioned PNTR(FstPtr:LstPtr, 0:MNOP), the logical array PPTY (FstPtr:LstPtr, 0:MNOP); and the real array DATA (1:MNOD, 0:MNOP)\(^7\). These arrays contain the dynamic part of the data structure. The parameters FstPtr, LstPtr, FstPpt, LstPpt, MNOD, and MNOP can be adapted by the user for his own purposes. In the actual implementation it is made particularly simple to adapt the parameters MNOD and MNOP. A small number of additional integers is necessary to handle the data structure. These numbers are needed as global variables. They are collected in the common block /DatGlh/ RtlLv, LstSpa, NOP, SzeX0, SzeY0, NrmOrd(4)\(^8\).

To reduce the number of arrays that should be available, in the FORTRAN implementation, the integer array Location is also stored into the integer array PNTR, such that Location(i,j) is found in PNTR([i-3,j], (i=0,1,2). The real arrays DCELL, DHWALL, DVWALL and DPOINT are all collected in the single real array DATA, of which the number of rows (MNOD) is to be determined by the user.

3.5 The PASCAL prototype

All essential algorithms with respect to the pointers and the properties in the data structure have been implemented in the PASCAL prototype that is given in this section.

In the main program of this prototype an example of their use has been given. An irregular domain is created. The coarsest grid is refined and, further, this fine grid is partly un-refined and other parts are refined again. The resulting data about the data structure are written to three files 'cel.cp', i=1,2,3, and the UNIX preprocessor 'grep' is used to draw the corresponding meshes. The resulting meshes are shown in the Figures 2 through 5.

\(\text{proorgan prototype (input,output)};\)

\(\text{\(^7\)MNOD: Maximum Number Of Data per patch; MNOP: Maximum Number Of Patches.}\)

\(\text{\(^8\)RtlLv = Root Level; LstSpa = Last Space; NOP = Number Of Patches; (SzeX0, SzeY0) denotes the number of cells on the zero level; NrmOrd = Normal Ordering.}\)
THE ACTIONS ON THE DATA STRUCTURE

{ -- created: November 1989 }{ -- this version: 1990-5-22 }{ -- author: P.W. Hemker }

const
MNOP = 2000;   { -- MaxNumberOfPatches }
MNOL = 20;     { -- MaxNumberOfLevels }
LNOL = -10;    { -- LowestNumberOfLevels }
nihil = 0;     { -- the nil pointer }
RootPointer = 1;

FirstPointer = 0; LastPointer = 8;
Parent=0; NE=1; SE=2; SW=3; NW=4; N=5; E=6; S=7; W=8;

FirstPpty = 1; LastPpty = 14;
Compile = 1; WallH = 2; WallV = 3;
BdyPoint = 4; BdyWallH = 5; BdyWallV = 6; BdyCell = 7;
GrnPoint = 8; GrnWallH = 9; GrnWallV = 10; GrnCell = 11;
Pregnant = 12; Sentenced = 13; Dead = 14;

type
    pointer = integer;
    order = array[1..4] of integer;
    string = array[1..24] of char;

var
    RootLevel, LastSpace, NumberOfPatches,
    RectSizeX, RectSizeY : integer;
    NormalOrder : order;

    PNTR : array [FirstPointer..LastPointer,.MNOP] of pointer;
    PPTY : array [FirstPpty..LastPpty,.MNOP] of boolean;
    Location : array [0..2,1..MNOP] of integer;

procedure error (i : pointer; s1, s2 : string);
begin { -- hard error message }
    writeln('fatal error'); writeln(i);
    writeln(s1); writeln(s2); writeln;
end;

procedure warning (i : pointer; s1, s2 : string);
begin { -- soft error message }
    writeln('warning ',i,' ',s1,' ',s2);
end;

procedure ReportIt (level: integer; filename: string);
var
    k,m : integer;
    scale : real;
    report:text;
begin
    scale := 1;
    for k := RootLevel to level-1 do scale := scale/2.0;
    rewrite(report,filename);

for m:= 1 to NumberOfPatches do
  if Location[0,m] = level then
    begin
      for k:= FirstPpty to LastPpty do
        if PPTY[k,m] then write(report,'T ') else write(report,'? ');
        write(report,m,' ');
        for k:= 1 to 2 do write(report,Location[k,m]=scale,' ');
        writeln(report, scale/2);
      end;
    end;
  end;

procedure Show (patch: pointer);
var
  k : integer;
  str: array[1..2] of char;
begin
  write(patch:3, ' @');
  for k:= 0 to 2 do write(' ', Location[k,patch]:3);
  write(' ');
  for k:= FirstPointer to LastPointer do write(PNTR[k,patch]:3);
  for k:= FirstPpty to LastPpty do
    begin if PPTY[k,patch] then
      case k of
        8: str:= 'gP';  9: str:= 'gH'; 10: str:= 'gV'; 11: str:= 'gC';
      end else str:= '___';
      writeln;
    end;
end;

function TwoPow ( k: integer ) : integer;
var { -- computes 2**k }
  1,p : integer;
begin
  p:= 1;
  if k>0 then for l:= 1 to k do p:= 2*p
  else if k<0 then error(nihil,'negative argument', 'TwoPow');
  TwoPow:= p;
end;

procedure InizData ;
var { -- initialization of the data structure }
  i : integer;
begin
  { -- makes a pointer of the nil pointer the nil pointer! }
  for i:= FirstPointer to LastPointer do PNTR[i,0]:= 0;
  for i:= FirstPpty to LastPpty do PPTY[i,0]:= false;

  NormalOrder[1]:= 3; NormalOrder[2]:= 4;
  NormalOrder[3]:= 2; NormalOrder[4]:= 1;
  NumberOfPatches:= 0; LastSpace:= 1;
end;
function MakePatch (k,i,j : integer; daddy : pointer): pointer;
{ -- Takes only care of Location and the parent pointer. -- }
var
  ii : integer;
begin
  LastSpace := LastSpace-1;
  repeat LastSpace := LastSpace+1
    until PPTY[Dead,LastSpace] or (LastSpace > NumberOfPatches);
  if LastSpace > NumberOfPatches then NumberOfPatches := LastSpace;

  if NumberOfPatches > MAXP then
    error(daddy, 'too many patches', 'MakePatch')
  else
    MakePatch := LastSpace;

{ -- NO properties, NO kids, NO neighbours, a FATHER }
for ii := FirstPointer to LastPointer do
  PPTY[ii,LastSpace] := nilill;
for ii := FirstPpty to LastPpty do
  PPTY[ii,LastSpace] := false;
  PNTR[Parent,LastSpace] := daddy;
Location [0,LastSpace] := k;
Location [1,LastSpace] := i;
Location [2,LastSpace] := j;
end;

function NewKid (wind : integer; daddy : pointer): integer;
{ -- Takes only care of Location and the parent pointer. -- }
var
  i,ii,ji,jj,k : integer;
begin
  if PNTR[wind, daddy] = nilill then
    { -- only if the kid doesn't yet exist! -- }
  begin
    k := Location[0,daddy];
    i := Location[1,daddy];
    j := Location[2,daddy];
    case wind of
      NE : begin ii := 2*i+1; jj := 2*j+1 end;
      SE : begin ii := 2*i+1; jj := 2*j end;
      NW : begin ii := 2*i ; jj := 2*j+1 end;
      SW : begin ii := 2*i ; jj := 2*j end;
    end;
    PNTR[wind, daddy] := MakePatch(k+1,ii,jj, daddy);
  end;
  NewKid := PNTR[wind, daddy];
end;

procedure GhostTies ( daddy: pointer);
{ -- This routine constructs the pointers between all existing
  -- kid-patches of daddy and their neighbours (the cousins).
  -- It assumes that neighbor relations already exist between
  -- daddy and his neighbours (the existing uncles).}
-- Routine ment for level k < 0

var
NEk, SEk, SWk, NWk :pointer; { -- the kids }
Nc, Ec, Sc, Wc :pointer; { -- the cousins }

begin
{ -- give all the kids their names }

NEk := PTR[ NE, daddy ];
SEk := PTR[ SE, daddy ];
NWk := PTR[ NW, daddy ];
SWk := PTR[ SW, daddy ];

{ -- first construct brother pointers }
if NEk <> SEk then begin PTR[ S, NEk ] := SEk; PTR[ N, SEk ] := NEk end;
if NWk <> SWk then begin PTR[ W, NWk ] := SWk; PTR[ NW, SWk ] := NWk end;
if NEk <> NWk then begin PTR[ W, NEk ] := NWk; PTR[ E, NWk ] := NEk end;
if SEk <> SWk then begin PTR[ W, SEk ] := SWk; PTR[ E, SWk ] := SEk end;

{ -- now construct cousin pointers }
if NEk <>=nil then begin

Nc := PTR[ SE, PTR[ N, daddy ]];
Ec := PTR[ NW, PTR[ E, daddy ]];
if Nc<>0 then begin PTR[ N, NEk ] := Nc; PTR[ S, Nc ] := NEk end
else varying(NEk, 'has no N-wall', 'GhostTies');
if Ec<>0 then begin PTR[ E, SEk ] := Ec; PTR[ W, Ec ] := SEk end
else varying(NEk, 'has no S-wall', 'GhostTies');

end;

if SEk <>=nil then begin

Sc := PTR[ NE, PTR[ S, daddy ]];
Ec := PTR[ SW, PTR[ E, daddy ]];
if Sc<>0 then begin PTR[ S, SEk ] := Sc; PTR[ N, Sc ] := SEk end;
if Ec<>0 then begin PTR[ E, SEk ] := Ec; PTR[ W, Ec ] := SEk end
else varying(SEk, 'has no S-wall', 'GhostTies');

end;

if NWk <>=nil then begin

Nc := PTR[ SW, PTR[ N, daddy ]];
Wc := PTR[ NE, PTR[ W, daddy ]];
if Nc<>0 then begin PTR[ N, NWk ] := Nc; PTR[ S, Nc ] := NWk end
else varying(NWk, 'has no N-wall', 'GhostTies');
if Wc<>0 then begin PTR[ W, NWk ] := Wc; PTR[ E, Wc ] := NWk end;

end;

if SWk <>=nil then begin

Sc := PTR[ NW, PTR[ S, daddy ]];
Wc := PTR[ SE, PTR[ W, daddy ]];
if Sc<>0 then begin PTR[ S, SWk ] := Sc; PTR[ N, Sc ] := SWk end;
if Wc<>0 then begin PTR[ W, SWk ] := Wc; PTR[ E, Wc ] := SWk end;

end;
procedure FamilyTies (daddy: pointer);
{ -- This routine constructs the pointers between all existing
  -- kids-patches of daddy and their the cousins at the NE-nodes.
  -- The pointers to the other sides (if necessary) already exist!
  -- It assumes that neighbour relations already exist between
  -- daddy and his neighbours (the existing uncles).
  -- Routine ment for levelk >= 0 }

var
  NEk, SEk, SWk, NWk, { -- the kids }
  NEEc, SEEc, NWcE, NEc, { -- far cousins }
  NUncle, EUncle, NEUncle { -- the uncles } : pointer;

begin
  if not PNTR[NE,daddy]<>0 then
    error(daddy,'incomplete','FamilyTies')
  else if PNTR[NE,daddy]<>0 then { -- all kids should exist }
    begin
      { -- give all the kids their names }
      NEk := PNTR[NE,daddy];
      SEk := PNTR[SE,daddy];
      NWk := PNTR[NW,daddy];
      SWk := PNTR[SW,daddy];
      if NEk*SEk*NWk*SWk = 0 then
        error(daddy,'kids missing','FamilyTies') else
      begin
        { -- first construct brother pointers }
        PNTR[S,NEk] := SEk; PNTR[N,SEk] := NEk;
        PNTR[S,NWk] := SWk; PNTR[N,SWk] := NWk;
        PNTR[W,NEk] := NWk; PNTR[E,NWk] := NEk;
        PNTR[W,SEk] := SWk; PNTR[E,SWk] := SEk;

        { -- give the uncles their names }
        NUncle := PNTR[N, daddy];
        EUncle := PNTR[E, daddy];
        NEUncle := PNTR[N,EUncle];
        if PNTR[E,NUncle] <> NEUncle then
          error(daddy,'inconsistent NEcorner','FamilyTies');

        { -- give the cousins their names }
        NEEc := PNTR[SE, NUncle];
        SEEc := PNTR[NW, EUncle];
        NWEc := PNTR[SW, NUncle];
        NEc := PNTR[SW, NEUncle];
        if NEc*NEEc*SEEc*NWBc*NEc = 0 then
          error(daddy,'nil neighbours','FamilyTies');

        { -- now construct cousin pointers }
        PNTR[N, NEk] := NEc ; PNTR[S,NEEc] := NEk;
        PNTR[E, NEk] := NEc ; PNTR[W,NEEc] := NEk;
        PNTR[E, SEk] := SEEc ; PNTR[W,SEEc] := SEE;
        PNTR[N, NWk] := NWc ; PNTR[S,NWC] := NWk;
        PNTR[E,NWC] := NWc ; PNTR[W,NWC] := NWc;
procedure BoundaryProperties(patch :pointer);
{ -- Determines either boundary walls and boundary cells
   -- around a complete patch (or level zero), or
   -- it determines the green walls and green cells
   -- around a complete patch (or a positive level)
   -- Boundary pties are permanent, they are obtained either on
   -- level 0 or by inheritance. Green pties may change any time. -- }
var
  NEnb, Nnb, Enb, Snb, Wnb, p :pointer;
  XxxPoint, XxxWallH, XxxWallV, XxxCell, i :integer;
begin
  if patch = nil then \s{ -- skip }\ else
  begin
    if Location[0,patch]<0 then
      error(patch,'No boundaries','BoundaryProperties')
    else if Location[0,patch]=0 then
      begin
        XxxWallH := BdyWallH; XxxPoint:= BdyPoint;
        XxxWallV := BdyWallV; XxxCell := BdyCell;
      end
    else
      begin
        XxxWallH := GrnWallH; XxxPoint:= GrnPoint;
        XxxWallV := GrnWallV; XxxCell := GrnCell;
      end;
  \s{ -- Give some neighbours their name \--}
  Nnb := PTR[N,patch];
  Enb := PTR[E,patch];
  NEnb := PTR[E, Wnb];
  Snb := PTR[S,patch]; \s{ -- possibly niln ! ! }\}
  Wnb := PTR[W,patch]; \s{ -- possibly niln ! ! }\}
if PPTY[Complete,patch] then
begin
  \s{ -- First determine the boundary Walls around a cell \--}
  PPTY[XxxWallH,patch]:= not PPTY[Complete,Snb];
  PPTY[XxxWallV,patch]:= not PPTY[Complete,Wnb];
  PPTY[XxxWallH, Wnb]:= not PPTY[Complete,Nnb];
  PPTY[XxxWallV, Enb]:= not PPTY[Complete,Enb];

  if PPTY[BdyWallH,patch] then PPTY[GrnWallH,patch]:= false;
  if PPTY[BdyWallV,patch] then PTTY[GrnWallV,patch]:= false;
  if PPTY[BdyWallH, Wnb] then PPTY[GrnWallH, Wnb]:= false;
  if PPTY[BdyWallV, Enb] then PPTY[GrnWallV, Enb]:= false;

  \s{ -- Secondly, determine the boundary Points around a cell \--}
PPTY[XxxPoint,patch] :=
    PPTY[XxxWallH,patch] or PPTY[XxxWallV,patch] or
    PTTY[XxxWallH, Wnb] or PPTY[XxxWallV, Snb];

PPTY[XxxPoint, Wnb] :=
    PPTY[XxxWallH, PNTR[W,Wnb]] or
    PTTY[XxxWallV,patch] or
    PPTY[XxxWallH, Wnb] or PPTY[XxxWallV, Wnb];

PPTY[XxxPoint, Eb] :=
    PPTY[XxxWallV, PNTR[S,Eb]] or
    PTTY[XxxWallH,patch] or
    PTTY[XxxWallH, Eb] or PPTY[XxxWallV, Eb];

PPTY[XxxPoint, NE] :=
    PTTY[XxxWallH, Nnb] or PPTY[XxxWallV, Eb] or
    PTTY[XxxWallH, NE] or PPTY[XxxWallV, NE];

{ -- Finally, determne the boundary cells around the patch } 

PPTY[XxxCell,patch] :=
    PPTY[XxxWallH,patch] or PPTY[XxxWallH,PNTR[N,patch]]
    or PPTY[XxxWallV,patch] or PPTY[XxxWallV,PNTR[E,patch]]; 

for i := N to W do 
    begin 
    p := PNTR[i,patch]; if p <> nil then
    begin
        PPTY[XxxCell, p] := PTTY[Complete, p] and
        ( PTTY[XxxWallH, p] or PTTY[XxxWallH, PNTR[N, p]]
        or PPTY[XxxWallV, p] or PTTY[XxxWallV, PNTR[E, p] ] );
    end;
    end 

else { -- the patch is NOT complete -- }
    begin 
    PTTY[XxxCell,patch] := false;
    PTTY[XxxWallH,patch] := PTTY[Complete,Snb];
    PTTY[XxxWallV,patch] := PTTY[Complete,Wnb];

    if PPTY[BdyWallH,patch] then PPTY[GrnWallH,patch] := false;
    if PPTY[BdyWallV,patch] then PPTY[GrnWallV,patch] := false;

    PPTY[XxxPoint,patch] := PTTY[XxxWallH, Wnb] or PTTY[XxxWallV, Snb] or
    PTTY[XxxWallH,patch] or PPTY[XxxWallV,patch];
    end;
end;
end;
end;
end;

procedure Offspring (daddy: pointer);
var 
    NEK, SEK, SWK, NWK, { -- the kids }
    UNcKle, EUNcKle, NEKcKle, { -- the uncles }
    NWK, VEN, NEK, SEE, NEc { -- the cousins } : pointer;
    w, k : integer;
begin
    if PTTY[Complete,daddy] then
    begin
        k := Location[0,daddy];
        if k < 0 then error(daddy,'negative level','Offspring');
        if PNTR[NE,daddy] <> 0 then
            error(daddy,'child already exists','Offspring');
3.5 The PASCAL prototype

SWk := NewKid(SW, daddy);
SEk := NewKid(SE, daddy);
NWk := NewKid(NW, daddy);
NEk := NewKid(NE, daddy);

{ -- give all the uncles their names }
NUncle := PNTR[ N, daddy];
EUncle := PNTR[ E, daddy];
NEUncle := PNTR[ N, EUncle];
if PNTR[ E, NUncle] <> NEUncle then
  error(daddy, 'inconsistent Wecorner', 'Offspring');

{ -- give the cousins their names }
NWN := NewKid(SW, NUncle);
NEN := NewKid(SE, NUncle);
NEE := NewKid(NW, EUncle);
SEE := NewKid(SW, EUncle);
NEn := NewKid(SW, NEUncle);

FamilyTiles(daddy);

{ -- take care of the major patch properties }
for w := NE to NW do
begin
  PPTY[Complete, PNTR[w, daddy]] := true;
  PPTY[ WallH, PNTR[w, daddy]] := true;
  PPTY[ WallV, PNTR[w, daddy]] := true;
end;
PPTY[ WallV, NEE ] := true; PPTY[ WallV, SEE ] := true;

{ -- take care of the boundary properties }
if PPTY[ BdyWallH, daddy ] then
begin
  PPTY[ BdyWallH, SEk ] := true; PPTY[ BdyPoint, SEk ] := true;
  PPTY[ BdyWallH, SWk ] := true; PPTY[ BdyPoint, SWk ] := true;
end;
if PPTY[ BdyWallV, daddy ] then
begin
  PPTY[ BdyWallV, NWk ] := true; PTTY[ BdyPoint, NWk ] := true;
  PPTY[ BdyWallV, SWk ] := true; PPTY[ BdyPoint, SWV ] := true;
end;
if PPTY[ BdyPoint, daddy ] then PTTY[ BdyPoint, SWk ] := true;

if PPTY[ BdyWallH, NUncle ] then
begin
end;
if PTTY[ BdyWallV, EUncle ] then
begin
  PTTY[ BdyWallV, SEE ] := true; PTTY[ BdyPoint, SEE ] := true;
  PTTY[ BdyWallV, NEE ] := true; PTTY[ BdyPoint, NEE ] := true;
end;
if PPTY[bdyCell, daddy] then
  begin
    if PPTY[bdyWallV, daddy] or PPTY[bdyWallH, daddy]
      then PPTY[bdyCell, SWk] := true;
    if PPTY[bdyWallV, daddy] or PPTY[bdyWallH, NUncle]
      then PPTY[bdyCell, NWk] := true;
    if PPTY[bdyWallH, daddy] or PPTY[bdyWallV, EUncle]
      then PPTY[bdyCell, SEk] := true;
    if PPTY[bdyWallV, EUncle] or PPTY[bdyWallH, NUncle]
      then PPTY[bdyCell, NEk] := true;
  end;
if PPTY[bdyPoint, EUncle] then PPTY[bdyPoint, NEc] := true;

{ -- take care of the green properties }
for w := NE to NW do BoundaryProperties(PNTR[w, daddy]);
end else:
  warning(daddy,'incomplete patch','Offspring');
end;

procedure HardRemovePatch(patch :pointer);
{ -- This routine removes a cell from the system. }
var
  i : integer;
  daddy : pointer;
begin
  { -- Pointers to the patch disappear -- }
  PNTR[S, PNTR[X, patch]] := nihil;
  PNTR[Y, PNTR[K, patch]] := nihil;
  PNTR[K, PNTR[S, patch]] := nihil;
  PNTR[E, PNTR[W, patch]] := nihil;

  { -- The relation with Parent is closed -- }
  daddy := PNTR[Parent, patch];
  for i := NE to NW do
    if PNTR[i, daddy] = patch then PNTR[i, daddy] := nihil;
    if patch < LastSpace then LastSpace := patch;

  { -- All pointers are removed -- }
  for i := FirstPointer to LastPointer do PNTR[i, patch] := nihil;

  { -- All other properties are removed -- }
  for i := FirstPpty to LastPpty do PPTY[i, patch] := false;
  PPTY[Dead, patch] := true;
end;

procedure RemovePatch(patch : pointer);
{ -- If possible, this routine removes a cell from the system. }
{ -- If a neighbouring cell needs this patch for its corner point
  -- it isn't possible. -- }
begin
  if not (PPTY[WallH, patch] or PPTY[WallV, patch]
    or PPTY[Complete, PNTR[S, PNTR[W, patch]]]
    or PPTY[Complete, PNTR[W, PNTR[S, patch]]]) then
3.5 The PASCAL prototype

HardRemovePatch(patch);
end;

procedure KillCell(patch : pointer);
{ -- Let the cell disappear from the system
   -- the patch possibly remains -- }
var
  Nnb, Enb, Snb, Wnb, NEnb : pointer;
begin
  if PPTY[Complete,patch] then
  begin
    if PNTR[NE,patch] <> nnil then
      error(patch,'has kids','KillCell');

    { -- Give the neighbours their name }
    Nnb := PNTR[N,patch];
    Enb := PNTR[E,patch];
    NEnb := PNTR[N,Enb]; if NEnb = nnil then NEnb := PNTR[E,Nnb];
    { -- they're possibly nil: }
    Snb := PNTR[S,patch];
    Wnb := PNTR[W,patch];

    if rot PPTY[WallIV,patch] then PPTY[bdyWallIV,patch] := false; { -- changed }
    if rot PPTY[WallIH,patch] then PPTY[bdyWallIH,patch] := false; { -- changed }
    { -- Cell disappears }
    PPTY[Complete,patch] := false;
    PPTY[Sentenced,patch] := false;
    PPTY[bdyCell,patch] := false;

    { -- Walls possibly disappear }
    PTTY[WallIV,patch] := PPTY[Complete,Nnb];
    PPTY[WallIH,patch] := PPTY[Complete,Snb];
    PPTY[WallIV,Enb] := PPTY[Complete,Enb];
    PPTY[WallIH,Nnb] := PPTY[Complete,Nnb];
    PPTY[WallIV,NEnb] := PPTY[Complete,Nnb] or PPTY[Complete,NEnb];
    PPTY[WallIH,NEnb] := PPTY[Complete,Enb] or PPTY[Complete,NEnb];

    { -- True boundary properties }
    PTTY[bdyWallIV,patch] := PTTY[bdyWallIV,patch] and PTTY[WallIV,patch];
    PTTY[bdyWallIH,patch] := PTTY[bdyWallIH,patch] and PTTY[WallIH,patch];
    PTTY[bdyWallIV,Enb] := PTTY[bdyWallIV,Enb] and PTTY[WallIV,Enb];
    PTTY[bdyWallIH,Nnb] := PTTY[bdyWallIH,Nnb] and PTTY[WallIH,Nnb];
    PTTY[bdyWallIV,NEnb] := PTTY[bdyWallIV,NEnb] and PTTY[WallIV,NEnb];
    PTTY[bdyWallIH,NEnb] := PTTY[bdyWallIH,NEnb] and PTTY[WallIH,NEnb];

    RemovePatch(patch); RemovePatch(Nnb);
    RemovePatch(Enb); RemovePatch(NEnb);

    BoundaryProperties(patch);
    BoundaryProperties(Nnb); BoundaryProperties(Enb);
    BoundaryProperties(NEnb);
    BoundaryProperties(Snb); BoundaryProperties(Wnb);
  end;
end;
procedure RemoveOffspring(patch :pointer);
{ -- If possible this routine removes the 4 kids of daddy and it
  -- adapts the datastructure correspondingly. It is only possible
  -- if all kids are sentenced. -- }
var
  w :integer;
begin
  if PPTY[Sentenced, PNTR[NE,patch]] and
      PPTY[Sentenced, PNTR[SE,patch]] and
      PPTY[Sentenced, PNTR[SW,patch]] and
      PPTY[Sentenced, PNTR[SW,patch]] then
    for w := NE to NW do KillCell( PNTR[w,patch]);
end;

procedure Scan (RootPointer: pointer; MyOrder :order;
  FromLevel, ToLevel :integer;
  procedure DoIt (patch :pointer) );
var
  lev,LowLevel :integer;
  IPT, i :array [MINL..MVL] of integer;
begin
  if RootPointer = nil then error(RootPointer,'nil pointer ','Scan');
  if ToLevel < FromLevel then error(RootPointer,'wrong ToLevels ','Scan');
  LowLevel := Location[0,RootPointer];
  if lowLevel > FromLevel then error(RootPointer,'wrong LowLevel','Scan');

  IPT[LowLevel] := RootPointer;
  if LowLevel = FromLevel then DoIt(RootPointer);

  if ToLevel > LowLevel then
    begin
      LowLevel := LowLevel + 1;
      for lev := LowLevel to ToLevel do i[lev] := 0;
      lev := LowLevel;
      repeat
        i[lev] := i[lev] + 1;
        if i[lev] <= 4 then
          begin
            IPT[lev] := PNTR[MyOrder[i[lev]], IPT[lev-1]];
            if IPT[lev] <> nil then
              begin
                if lev < ToLevel then
                  begin
                    if lev >= FromLevel then DoIt(IPTR[lev]);
                    lev := lev+1
                  end
                else
                  DoIt(IPTR[lev]);
              end
            end
          end
        end
      end
    end
end
3.5 The PASCAL prototype

```
    i[lev]:= 0;
    lev:= lev-1;
    end;
    until i[LowLevel] > 4;
    end;
end;

procedure RectCHV ( patch: pointer);
{ -- Auxiliary for GetRectangle -- }
var    i,j,k,RX, RY :integer;
begin
    k := Location[0,patch];
    i := Location[1,patch];
    j := Location[2,patch];
    RX:= RectSizeX;
    RY:= RectSizeY;
    if k <> 0 then error(patch,'non-zero level','RectCHV');
    if (i>RX) or (j>RY) then error(patch,'improper EECT ','RectCHV');
    if (i<RX) and (j<RY) then PPTY[Complete,patch]:= true;
    if i < RX then PPTY[WallH ,patch]:= true;
    if j < RY then PPTY[WallV ,patch]:= true;
end;

procedure RectKid ( patch: pointer);
{ -- Auxiliary for GetRectangle -- }
var    k,w,z,tt :integer;
begin
    k := Location[0,patch];
    if k=0 then error(patch,'positive level','RectKid');
    tt:= TwoPow(-1-k);
    for w:= 1 to 4 do
        begin
            z:= NewKid(NormalOrder[w],patch);
            if (tt*Location[1,z] > RectSizeX) or
               (tt*Location[2,z] > RectSizeY) then RemovePatch(z);
        end;
end;

procedure GetRectangle ( m,n: integer);
var    mm, mn, k :integer;
    RootPointer :pointer;
begin
    RectSizeX:= n;
    RectSizeY:= m;
    if (n<0) or (m<0) then
        error(nihil,'invalid input','GetRectangle');
    if n>m then mm:=n else mm:= m;
    RootLevel:= 0; nn:= 1;
    repeat
```
RootLevel := RootLevel-1;
nn := nn*2;
until nn > mm;

RootPointer := MakePatch(RootLevel, 0, C, nil);  
for k := RootLevel to -1 do  
  Scan(RootPointer, NormalOrder, k, k, RectKid);  
  { -- construct all pointers --}  
  for k := RootLevel to -1 do  
    Scan(RootPointer, NormalOrder, k, k, GhostTies);  
    { -- construct all CHV properties --}  
    Scan(RootPointer, NormalOrder, 0, 0, RectCHV);  
    { -- construct all Bdy properties --}  
    Scan(RootPointer, NormalOrder, 0, 0, BoundaryProperties);  
end;

procedure GetCylinder ( m, n: integer);  
var  
  Spointer, Npointer, Victim : pointer;  
i : integer;  
begin  
  GetRectangle(m, n);  
  Spointer := RootPointer;  
  while Location[0, Spointer] < 0 do  
    Spointer := PTR[SW, Spointer];  
  Npointer := Spointer;  
  while PTR[N, Npointer] <> nil do  
    Npointer := PTR[N, Npointer];  
  Victim := Npointer;  
  Npointer := PTR[S, Victim];  
  FPTY[Dead, Victim] := true;  
  HardRemovePatch(Victim);
  for i := 1 to RectSizeX do  
    begin  
      Victim := PTR[N, PTR[E, Npointer]];  
      FPTY[Dead, Victim] := true;  
      HardRemovePatch(Victim);
      PTR[S, Spointer] := Npointer;  
      PTR[N, Vpointer] := Spointer;  
      BoundaryProperties(Npointer);  
      Npointer := PTR[E, Npointer];  
      Spointer := PTR[E, Spointer];  
    end;
    PTR[S, Spointer] := Npointer;  
    PTR[N, Npointer] := Spointer;  
    BoundaryProperties(Npointer);  
    BoundaryProperties(Spointer);
end;

procedure CHKIRC (patch : pointer);  
  { -- This routine checks some consistency  
    -- of the neighbour pointers.  
  }
var  
  NW1, NW2, NE1, NE2, SW1, SW2, SE1, SE2 : pointer;
begin  
  NW1 := PTR[ N, PTR[ W, patch]];  
  NW2 := PTR[ W, PTR[ N, patch]};
if NW1 <> NW2 then writeln('CHKCIRC',patch,' ',NW1,' ',NW2);

SW1 := PNTR[ S, PNTR[ W, patch]];
SW2 := PNTR[ W, PNTR[ S, patch]];
if SW1 <> SW2 then writeln('CHKCIRC',patch,' ',SW1,' ',SW2);

NE1 := PNTR[ N, PNTR[ E, patch]];
NE2 := PNTR[ E, PNTR[ N, patch]];
if NE1 <> NE2 then writeln('CHKCIRC',patch,' ',NE1,' ',NE2);

SE1 := PNTR[ S, PNTR[ E, patch]];
SE2 := PNTR[ E, PNTR[ S, patch]];
if SE1 <> SE2 then writeln('CHKCIRC',patch,' ',SE1,' ',SE2);
end;

procedure CHKORDER (crd :order);
{ --This procedure checks the consistency
  -- of the order array.
}
begin
  if (crd[1] + ord[3] = 5) then else
  if (crd[1]=1) or (ord[3]=1) then else
  if (crd[2]=2) or (ord[4]=2) then else
  begin
    writeln(ord[1],' ',ord[2],' ',ord[3],' ',ord[4]);
    error(nihil,'wrong order','CHKORDER');
  end;
end;

var
  k,pp :integer;

begin { -- example main program -- }
  InizData;
  GetCylinder(5,5);

  writeln('level D');
  for k:= 1 to NumberOfPatches do Show(k);

  writeln('MAKE AN IRREGULAR FIGURE');
  KillCell(16);
  KillCell(21);
  KillCell(24);
  KillCell(27);
  KillCell(28);
  KillCell(29);
  KillCell(31);
  KillCell(39);
  KillCell(47);
  for k:= 1 to NumberOfPatches do Show(k);
write ln('Offspring, level 1');
scanf(RootPointer, NormalOrder, 0, 0, Offspring);
for k:=1 to NumberOfPatches do Show(k);

write ln('KILL OFFSPRING');
for k:=NE to NW do PPTY[Sentenced, PTR[k, 17]] := true;
RemoveOffspring(17);
for k:=NE to NW do PPTY[Sentenced, PTR[k, 18]] := true;
RemoveOffspring(18);
for k:=1 to NumberOfPatches do Show(k);

Offspring(119);
Offspring(103);
Offspring(101);
Offspring(106);
Offspring(111);
Offspring(105);
Offspring(107);
Offspring(110);
Offspring(109);
Offspring(017):

ReportIt(0, 'cell.cp0');
ReportIt(1, 'cell.cp1');
ReportIt(2, 'cell.cp2');
end.

In the figures 2-5, that follow on page 34, the data structure is shown that is generated as an example by the above main program. The numbers in the figures are patch numbers. Solid lines denote boundary walls, dotted lines internal walls. A green wall is identified by an arrow. Green cells are denoted by a G; boundary cells by a B; complete patches contain a C. Notice that boundary cells and green cells are always complete. (In the figures the B overprint the C.) Further, boundary points and green points are identified by B or G respectively.
4 The FORTRAN program

The routines that are to be used to handle the data structure are collected in the file 'basis.f'. The description of the important routines in this file is given in the following comment lines in Section 4.1. To use these routines it is also important to know the names of the global variables. These global variables are found in the include file 'basis.i'. This include file is to be included in each routine that makes use of the data structure. The text of the include file is given in Section 4.2.

4.1 The FORTRAN routines and variables

The following text describes the routines and variables available from the FORTRAN implementation. The text is part of the comment text in the actual program.

```c
The datastructure is kept in the arrays:

c integer PNTR (FstPtr:LstPtr, 0:MNOP) the pointers

c logical PPTY (FstPtr:LstPtr, 0:MNOP) the properties

c double precision DATA (1:MNOD, 0:MNOP) the data

c
The data are handled by the following subroutines:

c
A SUBROUTINE TO INITIALIZE THE DATA STRUCTURE

c subroutine DatIni(MNDFac, MNODac, PNTR, PPTY, DATA)

c A routine to be called once during a run, before the

data structure is used.

c MNOPac and MNODac define the actual MNOP and MNOD used.

c MNOP: Maximum Number Of Patches.

c MNOD: Maximum Number Of real Data per patch.

c (The arrays PNTR, PPTY and DATA should have at least the

c size specified above.)

c
SUBROUTINES FOR THE CONSTRUCTION OF A DOMAIN

c subroutine MkRec(m, n, PNTR, PPTY, DATA)

c This subroutine constructs a data structure corresponding to a

c rectangular domain, with (on level 0)

c m cells in the Xi-direction and

c n cells in the Eta-direction.

c For the relation between (Xi,Eta)-coordinates, cell numbering

c and physical coordinates we refer to [1].

c

c subroutine MkCyl(m, n, PNTR, PPTY, DATA)

c This subroutine constructs a data structure corresponding to a

c cylindrical domain, with (on level 0)

c m cells in the Xi-direction and

c n cells in the Eta-direction.

c The cylindrical domain is constructed such that the patches at
```
maximum Eta are the bottom neighbours of the patches at Eta equal zero.

subroutine RaCe10(i, j, PNTR, PPTY, DATA)
A subroutine to re-shape the domain on level 0. This subroutine is to be called for each cell that is to be removed from the rectangle or cylinder that was made by MkRec or MkCyl.
The cell is denoted by (i,j), the (Xi,Eta)-coordinates of the complete patch containing the cell.

SUBROUTINES TO ADD TO OR REMOVE FROM THE DATA STRUCTURE

subroutine MdOfsp(daddy, PNTR, PPTY, DATA)
A subroutine that creates four new cells as kids of the cell identified by the pointer 'daddy'.

subroutine RnOfsp(daddy, PNTR, PPTY, DATA)
A subroutine that removes the kid-cells of the cell identified by the pointer 'daddy', provided that all these kid-cells are marked as 'Sentenced'.

A SUBROUTINE TO SCAN THE PATCHES IN THE DATA STRUCTURE

subroutine Scan(RtPt, order, FrMLv, ToLv, DoIz, PNTR, PPTY, DATA)
A subroutine that scans all patches that are descendents of the patch denoted by the pointer 'RtPt', and that are on a level 'lv' for which 0 <= FrMLv <= lv <= ToLv.
The patches are visited tree-wise, in a sequence determined by 'order'.
At each patch visited a call is made to the subroutine 'DoIt':
call DoIt(patch, PNTR, PPTY, DATA)
where 'patch' identifies the patch visited.
The subroutine 'DoIt' can be any subroutine constructed by the user, provided that it has the above syntaxes.
If necessary, additional communication between the actual subroutine 'DoIt' and the (sub)program calling 'Scan' can be taken care of by a locally defined COMMON block, only known by the (sub)program calling 'Scan' and the actual 'DoIt'.

SOME ADDITIONAL SUBROUTINES

subroutine Error(integer, string1, string2)
A Subroutine called after a fatal error in the program occurred.
The integer and strings are printed on standard output.

subroutine Warnin(integer, string1, string2)
A subroutine called after a non-fatal error in the program occurred.
The integer and strings are printed on standard output.

subroutine Show(patch, PNTR, PPTY, DATA)
A subroutine to print the PNTR and PPTY data of the patch 'patch'.
These data are given as one line of output on the standard output.
4.1 The FORTRAN routines and variables

COMMON BLOCK FOR THE DATA STRUCTURE

common /DatGlbl/ MNOP, MNOD,
+ Rtlv, LastSpa, NOP, SizeX0, SizeY0, NrmOrd
The passing of global properties of the data structure to
subroutines is provided by the COMMON block 'DatGlbl'.

Meaning of some global parameters and variables:

VARIABLES IN THE COMMON BLOCK 'DatGlbl'

MNOP       Maximum Number Of Patches
MNOD       Number Of Data (of a patch)
Rtlv       Root Level
LastSpa    Last Space
NOP         Number Of Patches
SizeX0      rectangle Size Xi-direction on level 0
SizeY0      rectangle Size Eta-direction on level 0

POINTER PARAMETERS

Nil         Nil Pointer
RtPtr       Root Pointer

POINTER ARRAY BOUND PARAMETERS

FirstPtr    First Pointer
LastPtr     Last Pointer

POINTER ARRAY INDEX PARAMETERS

LV         Level
XX         Xi-coordinate
YY         Eta-coordinate
PT         Parent
NE         North-East (kid)
SE         South-East (kid)
SW         South-West (kid)
NW         North-West (kid)
NN         North
EE         East
SS         South
WW         West

PROPERTY ARRAY BOUND PARAMETERS

FirstPrt    First Property
LastPrt     Last Property

PROPERTY ARRAY INDEX PARAMETERS

Compl       Complete
WallH       Horizontal Wall
WallV       Vertical Wall
BdyPnt      Boundary Point
BdyWallH    Boundary Wall Horizontal
**A SCANNING ORDER**

This order used by 'Scan' causes the patches to be visited from the SW- to the NE-corner of the domain. In some subroutines using 'Scan', this scanning order is essential and therefore should not be changed.

**INCLUDE FILE**

In order to create a compact source-code file, the subroutines make use of an 'include file'. The way of including this file by a compiler directive such as 'include', depends on the machine and FORTRAN compiler being used. The FORTRAN-code becomes ANSI-compatible by once and for all copying the include file into the subroutines, at the location where it is now included.
4.2 The FORTRAN include file

The following text is the contents of the include file "basis.i", that is used to have common identifiers, with the same meaning, for all subroutines that make use of the data structure.

```fortran
integer     FatPtr, LetPtr,
            LV, XX, YY, PT,
            NE, SE, SW, NW, NN, EE, SS, WW,
            FstPtr, LetPtr,
            Compl, WallE, WallN,
            BdyPnt, BdyWaH, BdyWaV, BdyCel,
            GrnPnt, GrnWaH, GrnWaV, GrnCel,
            Prgnt, Sntncl, Dead,
            Nil, Rtptr
parameter    (FatPtr =-3, LetPtr = 8,
             LV  =-3, XX  =-2, YY  =-1, PT  = 0,
             NE  = 1, SE  = 2, SW  = 3, NW  = 4,
             NN  = 5, EE  = 6, SS  = 7, WW  = 8,
             FstPtr = 1, LetPtr =14,
             Compl = 1, WallH = 2, WallV = 3,
             BdyPnt = 4, BdyWaH = 5, BdyWaV = 6, BdyCel = 7,
             GrnPnt = 8, GrnWaH = 9, GrnWaV = 10, GrnCel = 11,
             Prgnt =12, Sntncl =13, Dead =14,
             Nil  = 0, Rtptr =1)

integer     MNOP, MNOD,
            Rtlv, LetSpa, NOP, SizeXC, SizeYO, NrmOrd(4)
common /DatGlb/     MNOP, MNOD,
                   Rtlv, LetSpa, NOP, SizeXC, SizeYO, NrmOrd

integer     PNTR(FatPtr:LetPtr, 0:MNOP)
logical     PTTY(FatPtr:LetPtr, 0:MNOP)
double precision DATA(1:MNOP, 0:MNOP)
```
Figure 2: The example (see page 28): all cells on level 0. See section 3.5.
Figure 3: The example: all cells on level 1. See section 3.5.
Figure 4: The example: all cells on level 2. See section 3.5.
4.2 The FORTRAN include file

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 |
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |

Figure 5: The example: all cells. See section 3.5.
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