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SPACE MAPPING AND DEFECT CORRECTION

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Space mapping, [1][2], is a technique to efficiently solve a difficult optimization problem

$$\mathbf{x}^* = \operatorname{argmin}_{\mathbf{x}\in X} \ \| \mathbf{f}(\mathbf{x}) - \mathbf{y} \| \ , \tag{1}$$

with $\mathbf{f} : X \subset \mathbb{R}^n \to \mathbb{R}^m$; $\mathbf{y} \in \mathbb{R}^m$; $m \geq n$, by means of the iterative use of a similar but simpler optimization problem with $\mathbf{c} : Z \subset \mathbb{R}^n \to \mathbb{R}^m$,

$$\mathbf{z}^* = \operatorname{argmin}_{\mathbf{z}\in Z} \ \| \mathbf{c}(\mathbf{z}) - \mathbf{y} \| \ . \tag{2}$$

We show that space-mapping optimization can be understood in the framework of defect correction [3]. Then, space-mapping algorithms can be seen as special cases of defect correction iteration. In order to analyze properties of space mapping we introduce the concept of flexibility of the underlying models. The best space-mapping results are obtained for so-called equally flexible models.

By introducing an affine operator $\mathbf{S} : \mathbf{c}(Z) \to \mathbf{f}(X)$, two models can be made equally flexible, at least in the neighborhood of a solution. This motivates the following improved space-mapping (or manifold-mapping) algorithm (with $\overline{\mathbf{p}} : X \to Z$ an arbitrary bijection):
(1) Set $k = 0$, set $S_0 = I$ the $m \times m$ identity matrix, and compute $\mathbf{x}_0 = \operatorname{argmin}_{\mathbf{x}\in X} \ \| \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x})) - \mathbf{y} \|$.
(2) Compute $\mathbf{f}(\mathbf{x}_k)$ and $\mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_k))$.
(3) If $k > 0$, with $\Delta\mathbf{c}_i = \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_{k-i})) - \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_k))$ and $\Delta\mathbf{f}_i = \mathbf{f}(\mathbf{x}_{k-i}) - \mathbf{f}(\mathbf{x}_k)$, $i = 1, \cdots, \min(n, k)$, we define ΔC and ΔF to be the rectangular $m \times \min(n, k)$-matrices with respectively $\Delta\mathbf{c}_i$ and $\Delta\mathbf{f}_i$ as columns. Their singular value decompositions are respectively $\Delta C = U_c \Sigma_c V_c^T$ and $\Delta F = U_f \Sigma_f V_f^T$. Now, $S_k = \Delta F \, \Delta C^\dagger + (I - U_\mathbf{f} \, U_\mathbf{f}^T) \, (I - U_\mathbf{c} \, U_\mathbf{c}^T)$ and the approximate affine mapping is $\mathbf{S}_k \, \mathbf{v} = \mathbf{f}(\mathbf{x}_k) + S_k(\mathbf{v} - \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_k)))$, $\forall \mathbf{v} \in Y$, which, for $l > 0$ and $l = k-1, \cdots, \max(0, k-n)$, satisfies $S_k \left(\mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_l)) - \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_k)) \right) = \mathbf{f}(\mathbf{x}_l) - \mathbf{f}(\mathbf{x}_k)$.
(4) The next iterand is computed as

$$\mathbf{x}_{k+1} = \operatorname{argmin}_{\mathbf{x}\in X} \ \left\| \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x})) - \mathbf{c}(\overline{\mathbf{p}}(\mathbf{x}_k)) + \left[\Delta C \ \Delta F^\dagger + I - U_c U_c^T \right] (\mathbf{f}(\mathbf{x}_k) - \mathbf{y}) \right\| ,$$

where † denotes the pseudo-inverse, $\Delta F^\dagger = V_f \Sigma_f^{-1} U_f^T$.

By a few simple examples we illustrate some phenomena analyzed.

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