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ABSTRACTS

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$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \mathbf{V} + \mathbf{g}, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla)T = \frac{1}{\rho C}\nabla \cdot (k\nabla T), \tag{3}$$

where $\mathbf{V}$ is the velocity vector; P , ρ and ν are the pressure, density and kinematic viscosity of the fluid, respectively, and $\mathbf{g}$ represents the body force per unit mass. T is the temperature, k is the material thermal conductivity and C is the heat capacity of the particle.

The function of the fluid volume [3] has been used to determine the region of the molten particle flattening, to define the interface location and to trace the movement of this interface. The scalar function F is defined whose value is equal to the fractional volume of the cell occupied by the fluid. F is assumed to be unity when the cell is fully occupied by the fluid and zero when the cell is empty. Cells with the values $0 < F < 1$ contain a free surface. The function F satisfies the conservation equation as follows

$$\frac{\partial F}{\partial t} + (\mathbf{V} \cdot \nabla)F = 0. \tag{4}$$

In the present stage of investigations the solution of the problem by means of the full Navier-Stokes equations is more complete in comparison with the previous stage, where a semi-empirical approach was used [2], which combined the use of the physical views about the interaction of the particle and substrate with empirical data and simplified the problem of investigation of plasma spraying. The new model can be extended to predict voids in thermal spray coatings.

For numerical calculations we created computational algorithms on the basis of the finite-difference method, which were realized in the form of a complex of applied programs.

We investigated the effects of some important processing parameters such as the impact velocity, droplet diameter, the pressure and temperature of plasma on the flattening and solidification of a single liguid particle. The pressure and temperature distribution in the particle and substrate at different spraying parameters were obtained.

References

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A DEFECT CORRECTION METHOD FOR PARABOLIC SINGULAR PERTURBATION PROBLEMS ON A RECTANGLE [3]

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On a rectangle we consider the Dirichlet problem for a singularly perturbed parabolic PDE of reaction-diffusion type with the perturbation parameter ε from the half-interval (0,1]. The solution of the problem

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has singularities such as parabolic boundary layers which are one-dimensional in the neighbourhood of the smooth boundaries and two-dimensional (corner) in the neighbourhood of the vertexes of the rectangle. For the boundary-value problem we know a special difference scheme (the base scheme), the solution of which converges ε-uniformly. This scheme is classical finite difference approximations of the differential equation on piecewise uniform meshes refined in the neighbourhood of the boundary layers. Its solution converges with the order of accuracy $O(N^{-2}\ln^2 N + N_0^{-1})$, where $N = \min[N_1, N_2]$, and $N_0 + 1$, $N_i + 1$ is the number of the nodes, respectively, in the time mesh and in the space mesh along the axis x_i, $i = 1, 2$. For the problem stated above it is of great interest to develop difference schemes with a higher order of convergence.

In this work we develop special ε-uniformly convergent schemes for which the order of accuracy is more than one with respect to the time variable. For solving the problem, we adapt the system of discrete problems (difference schemes) constructed using the base scheme. The discrete problems in the system are solved sequentially. These problems are constructed by such a way that the grid solutions (and its derivatives) already obtained are used then in the correction procedure to increase the consistency order of the next discrete problem. This correction method allows us to find the approximate solution with high-order time-accuracy, uniform in ε. To achieve ε-uniform convergence, we use a grid with nodes that are condensed in the neighbourhood of the boundary layer.

We present the results of numerical experiments that confirm the efficiency of the difference schemes developed.

CONSTRUCTION OF OPERATOR INTERPOLATION FORMULAS BY MEANS OF FOURIER-BESSEL INTEGRAL TRANSFORM

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Let $X = X(\ [0,2\pi] \times [0,\infty)\ )$ be the set of functions for which the Fourier-Bessel integral exists, and $F(x) \equiv F(t;x)$ $(t \in T \subseteq R^N)$ be an operator mapping X into Y , where Y is the space of functions defined on the set T.

The interpolation formulas $L_n(F;t,x) \equiv L_n(x) : X \to Y$, such that the following conditions of interpolation

$$L_n(x_k) \equiv F(x_k),\ k = \overline{0,n}$$

are satisfied, for differentiable in the sense of the Gateaux and nondifferentiable operators defined on the set X by means of Fourier-Bessel integral transform have been constructed. Such formulas are invariant with respect to the following operator polynomials of the order n

$$A_n(t;x) = a_0(t) + \sum_{k=1}^{n} \int\limits_{T_k} \int\limits_{R_+^k} a_k(t;\tilde{s}_k,\tilde{\theta}_k) \prod_{j=1}^{k} \frac{\partial^{\alpha_j+\beta_j}}{\partial s_j^{\alpha_j} \partial \theta_j^{\beta_j}} x(s_j,\theta_j) d\tilde{s}_k d\tilde{\theta}_k,$$

where $\tilde{s}_k = \{s_j\}_{j=1}^k$, $\tilde{\theta}_k = \{\theta_j\}_{j=1}^k$, $d\tilde{s}_k = ds_1 ds_2 \ldots ds_k$, $d\tilde{\theta}_k = d\theta_1 d\theta_2 \ldots d\theta_k$; $a_0(t)$ and $a_k(t;\tilde{s}_k,\tilde{\theta}_k)$ are arbitrary given functions; $x \in X$, $t \in T$, $s_j \in [0,2\pi]$, $\theta_j \in [0,\infty)$, $T_k = [0,2\pi]^k$; $\alpha_j, \beta_j = 0,1,\ldots$; $j = \overline{0,k}$; $k = \overline{1,n}$.

In particular, the formulas of linear interpolation have the form

$$L_1(x) = F(x_0) + \frac{1}{2\pi} \int\limits_{T_1} \int\limits_{R_+} [x(\tau,\lambda) - x_0(\tau,\lambda)] \delta F[x_0(\cdot,\cdot) + g(\tau,\lambda,\cdot,\cdot;x_1 - x_0); \rho(\tau,\lambda,\cdot,\cdot)] d\tau d\lambda,$$

and

$$L_1(x) = F(x_0) + \frac{1}{2\pi} \int\limits_{T_1} \int\limits_{R_+} \frac{x(\tau,\lambda) - x_0(\tau,\lambda)}{x_1(\tau,\lambda) - x_0(\tau,\lambda)} d_\tau d_\lambda F[x_0(\cdot,\cdot) + g(\tau,\lambda,\cdot,\cdot;x_1 - x_0)]$$