Optimisation in electromagnetics with the space-mapping technique

D. Echeverría and D. Lahaye
Centrum voor Wiskunde en Informatica (CWI), Amsterdam, The Netherlands
L. Encica
Eindhoven University of Technology (TU/e), Eindhoven, The Netherlands, and
P.W. Hemker
Centrum voor Wiskunde en Informatica (CWI), Amsterdam, The Netherlands

Abstract

Purpose – Optimisation in electromagnetics, based on finite element models, is often very time-consuming. In this paper, we present the space-mapping (SM) technique which aims at speeding up such procedures by exploiting auxiliary models that are less accurate but much cheaper to compute.

Design/methodology/approach – The key element in this technique is the SM function. Its purpose is to relate the two models. The SM function, combined with the low accuracy model, makes a surrogate model that can be optimised more efficiently.

Findings – By two examples we show that the SM technique is effective. Further we show how the choice of the low accuracy model can influence the acceleration process. On one hand, taking into account more essential features of the problem helps speeding up the whole procedure. On the other hand, extremely simple auxiliary models can already yield a significant acceleration.

Research limitations/implications – Obtaining the low accuracy model is not always straightforward. Some research could be done in this direction. The SM technique can also be applied iteratively, i.e. the auxiliary model is optimised aided by a coarser one. Thus, the generation of hierarchies of models seems to be a promising venue for the SM technique.

Originality/value – Optimisation in electromagnetics, based on finite element models, is often very time-consuming. The results given show that the SM technique is effective for speeding up such procedures.

Keywords Magnetic devices, Optimization techniques, Finite element analysis

Paper type Research paper

1. Introduction

In this paper, we give a brief exposition of the space-mapping (SM) technique and we illustrate this by two electromagnetic design problems.

SM (Bandler *et al.*, 1994, 2004) is a computational technique of defect-correction type (Böhmer *et al.*, 1984): having available an efficient but imperfect method that solves a problem only by approximation, one can improve its performance by measuring the

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defect in performance and modifying the data for the problem accordingly, such that a repeated use of the same imperfect method gives a better, and, finally, perfect result.

In fact the method works like shooting with a crooked gun. The first shot, properly aimed at the target will miss. Then we measure – or guess – the distance between the target and the spot hit by the bullet. Taking into account a compensation for the deviation, the target can be hit by the same gun if the point aimed at is adapted. If the second shot is not yet perfect, the procedure can be iterated.

This is a well known and often used principle in numerical computations. Examples are Newton's iteration for the solution of non-linear equations or relaxation methods or preconditioning schemes for the solution of linear systems.

Section 2 explains the SM principle and introduces the most popular algorithm. In Section 3 several minimisation techniques are compared by means of two electromagnetic test problems from literature (Choi *et al.*, 2001; Saldanha *et al.*, 1992). In both cases the best results are obtained with the SM technique.

2. Space-mapping basics

Let the specifications for the optimisation problem be denoted by $\mathbf{y} \in \mathbb{R}^m$. We assume that the behaviour of the system can be mathematically described in two different ways. First, by an *accurate* but *expensive to compute* model. This is the *fine model* and is denoted by $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^m$, where $\mathbf{x} \in X \subset \mathbb{R}^n$ is the *control variable*. A finite element solution of the field equations is usually taken as a fine model. The optimisation problem that finally should be solved, consists of minimising over X the cost function $F(\mathbf{x}) = \|\mathbf{f}(\mathbf{x}) - \mathbf{y}\|$, which measures the discrepancy between the specifications and the response of that precise mathematical model. For simplicity, we first assume that a unique optimum exists. It will be denoted by $\mathbf{x}^*$. But the method works also in cases when there is a manifold of optima or if the minimum should be reached within some tolerance, so that solutions are found in a small subregion of X. The cost function is assumed to be continuous but derivative information is considered unavailable.

SM needs a second model, *less accurate* than the fine one but much *cheaper* to compute. It is called the *coarse model* and is denoted by $\mathbf{c}(\mathbf{z}) \in \mathbb{R}^m$, with $\mathbf{z} \in Z \subset \mathbb{R}^n$ the associated control variable. This model can be optimised over Z by introducing the cost function $C(\mathbf{z}) = \|\mathbf{c}(\mathbf{z}) - \mathbf{y}\|$. As with the fine model, we assume that a unique optimum exists, which is denoted by $\mathbf{z}^*$. The coarse model should be such that it can be optimised efficiently by some computational method, but – of course – the solution of this coarse problem only yields a (very) rough approximation for the original problem.

In principle the spaces X and Z can be different because of the different formulations of the same problem. However, in most cases, as in our experiments, the control variables $\mathbf{x}$ and $\mathbf{z}$ refer to the same physical entities. Nevertheless the notation with different X and Z is still useful because it emphasises the fact that the respective models and their solutions are different, and that optimisation over Z is a much simpler problem than over X.

The main idea underlying SM consists of two parts: first, *optimising the coarse model* and then *mapping* the coarse optimum $\mathbf{z}^*$ into the fine control parameter space X. This mapping between the control parameter spaces X and Z is called the *SM function*. The SM function $\mathbf{p} : X \rightarrow Z$ is defined by

$$\mathbf{p}(\mathbf{x}) = \underset{\mathbf{z}\in Z}{\operatorname{argmin}} \|\mathbf{c}(\mathbf{z}) - \mathbf{f}(\mathbf{x})\|. \tag{1}$$

Again existence and uniqueness is assumed. In practice multiple optima can be present. Several modifications of the SM function are suggested in Bandler *et al.* (2004) so as to alleviate this problem. The quantity $\|\mathbf{c}(\mathbf{z}) - \mathbf{f}(\mathbf{x})\|$ is just a measure of the similarity between the response of the two models. This is shown in Figure 1 for a fine and coarse model with only a single control variable. The example is a one-parameter version (only $\mathbf{x}_2$) of the problem in Section 3.2, with coarse models denoted by $(N_m, N_c) = (16, 8)$, (8,4) or (6,2). They are ordered with respect to their proximity to the fine model, (16,8) being the closest of the three. The straight line (the identity function) would represent equal responses for the fine and the coarse models.

A SM function $\mathbf{p}$ is called a *perfect mapping* iff

$$\mathbf{z}^* = \mathbf{p}(\mathbf{x}^*), \tag{2}$$

i.e. if it maps the true solution to the optimal solution of the coarse problem. Although this property is by no means guaranteed, it will be clear that it is often true by approximation since in most practical cases we either have $\mathbf{f}(\mathbf{x}^*) \simeq \mathbf{c}(\mathbf{z}^*)$ or $\mathbf{f}(\mathbf{x}^*) \simeq \mathbf{y}$. If we rewrite equation (2)

$$\underset{\mathbf{z}\in Z}{\operatorname{argmin}} \|\mathbf{c}(\mathbf{z}) - \mathbf{y}\| = \underset{\mathbf{z}\in Z}{\operatorname{argmin}} \|\mathbf{c}(\mathbf{z}) - \mathbf{f}(\mathbf{x}^*)\|, \tag{3}$$

it can be easily seen how the two commented facts imply the *approximate* perfect mapping $\mathbf{p}(\mathbf{x}^*) \simeq \mathbf{z}^*$.

We can choose between two related but different approaches to realise SM. The first one expects $\mathbf{x}_p^*$, the solution of

$$\mathbf{x}_p^* = \underset{\mathbf{x}\in X}{\operatorname{argmin}} \|\mathbf{p}(\mathbf{x}) - \mathbf{z}^*\|, \tag{4}$$

to be a (very) good approximation to $\mathbf{x}^*$, the solution wanted. Hence, we expect our solution $\mathbf{x}^*$ to approximately satisfy equation (2), where $\mathbf{z}^*$ is easy to compute. This leads to the so-called *original* SM (Bandler *et al.*, 2004).

The other approach is dual and based on the replacement of the expensive fine model optimisation by a cheap surrogate model one. For this purpose we can take the coarse model and, to improve its accuracy, combine it with the SM function. Because of equation (1) we expect that $\mathbf{c}(\mathbf{p}(\mathbf{x})) \simeq \mathbf{f}(\mathbf{x})$. So it is reasonable that the minimisation of $\|\mathbf{c}(\mathbf{p}(\mathbf{x})) - \mathbf{y}\|$ will give us a value close to the desired optimum $\mathbf{x}^*$:

$$\mathbf{x}_d^* = \underset{\mathbf{x}\in X}{\operatorname{argmin}} \|\mathbf{c}(\mathbf{p}(\mathbf{x})) - \mathbf{y}\|. \tag{5}$$

This leads to the *new* SM approach (Søndergaard, 1999).

These two forms show close similarity with two forms of the defect correction processes known from numerical analysis (Böhmer *et al.*, 1984). It should be noted that if $\mathbf{z}^* \in \mathbf{p}(X)$, we have $\mathbf{p}(\mathbf{x}_p^*) = \mathbf{p}(\mathbf{x}_d^*)$ and with an injective $\mathbf{p}$ that $\mathbf{x}_p^* = \mathbf{x}_d^*$. With a

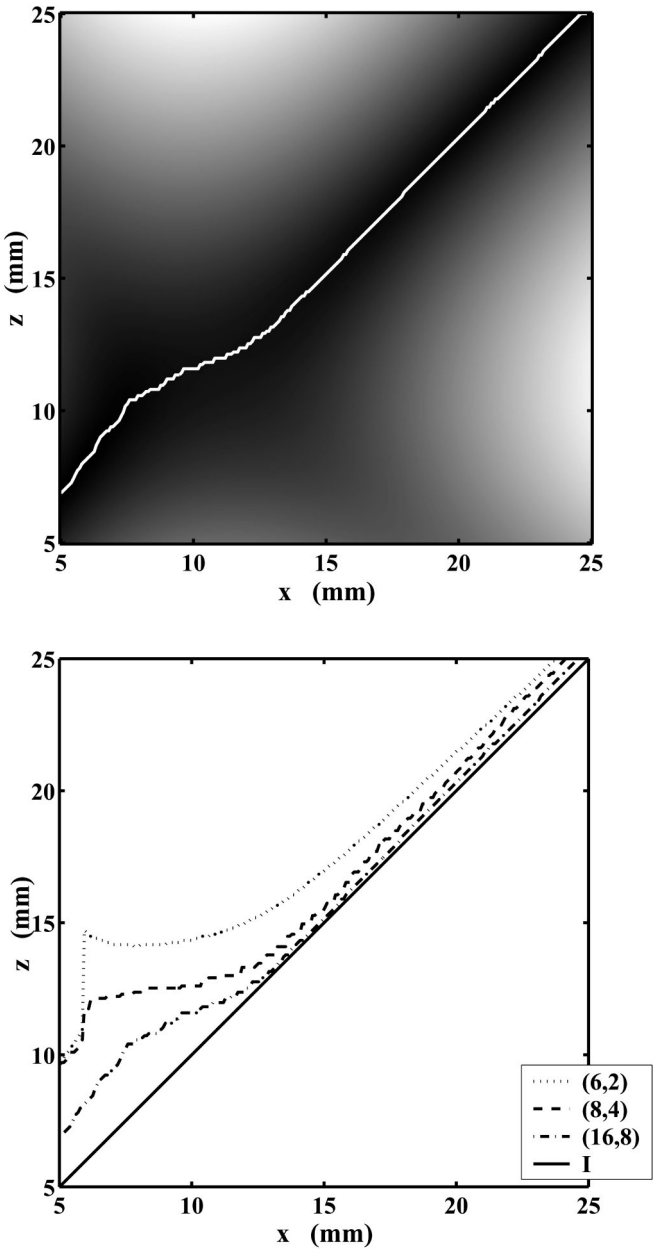

Figure 1.
Illustration of the SM function. The figure at the top shows a density-plot (the darker, the smaller value) of the function $\|\mathbf{c}(\mathbf{z}) - \mathbf{f}(\mathbf{x})\|$ for a simplified version of the example in Section 3.2. The minimum in $\mathbf{z}$ for each $\mathbf{x}$ corresponds with the associated SM function (white line). The figure at the bottom shows three SM functions, for several coarse models in the same problem.

perfect mapping, we even have $\mathbf{p}(\mathbf{x}_p^*) = \mathbf{p}(\mathbf{x}_d^*) = \mathbf{p}(\mathbf{x}^*)$. In general we cannot assure injectivity for $\mathbf{p}$ but usually in practice this property is held in an approximate way[1]. Thus, we can expect that $\mathbf{x}_p^* \simeq \mathbf{x}_d^* \simeq \mathbf{x}^*$, i.e. both the original and new approaches will lead us to the proximity of the desired optimum.

The SM function is expensive to evaluate since it implies the use of the fine model. Both SM approaches, when implemented in an iterative scheme (see Figures 2 and 3), look for efficient approximations $\mathbf{p}_k$ to the SM function. An often used linear $\mathbf{p}_k$ proposed is (Søndergaard, 1999),

$$\mathbf{p}_k(\mathbf{x}) = \mathbf{p}(\mathbf{x}_k) + \mathbf{B}_k(\mathbf{x} - \mathbf{x}_k),$$

where $\mathbf{B}_k$ is a Broyden-type approximation (Broyden, 1965) to the Jacobian of the mapping function updated with

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \frac{\mathbf{p}(\mathbf{x}_{k+1}) - \mathbf{p}(\mathbf{x}_k) - \mathbf{B}_k\mathbf{h}_k}{\mathbf{h}_k^T\mathbf{h}_k}\mathbf{h}_k^T,$$

being $\mathbf{h}_k = \mathbf{x}_{k+1} - \mathbf{x}_k$.

Combining this linearisation with the original SM algorithm, the popular *aggressive space mapping* (*ASM*) (Bandler *et al.*, 1995) is obtained (Figure 4). ASM assumes $\mathbf{z}^* \in \mathbf{p}(X)$ and just solves equation (4) by a quasi-Newton iteration with an approximate Jacobian. This is a convenient choice because only one evaluation of the fine model is needed per iteration. For a constrained minimisation ASM should be adapted because in its present form it cannot be assured that $\mathbf{x}_k$ is really the minimum of $\|\mathbf{p}_k(\mathbf{x}) - \mathbf{z}^*\|$ over X. In the experiments reported in this paper, all approximate values satisfy $\mathbf{x}_k \in X$, so the ASM was applied as it appears in Figure 4.

$\mathbf{x}_0 = \mathbf{z}^* = \underset{\mathbf{z}\in Z}{\text{argmin}} \|\mathbf{c}(\mathbf{z}) - \mathbf{y}\|;$
$\mathbf{p}_0 = I;$
for $k = 0, 1, \ldots,$
do
$\quad \mathbf{x}_{k+1} = \underset{\mathbf{x}\in X}{\text{argmin}} \|\mathbf{p}_k(\mathbf{x}) - \mathbf{z}^*\|;$
$\quad$ compute updated $\mathbf{p}_{k+1}$ by means of $\{\mathbf{x}_j\}_{j=0}^{k+1};$
enddo;

Figure 2.
The SM algorithm associated with the original approach

$\mathbf{x}_0 = \mathbf{z}^* \ \underset{\mathbf{z}\in Z}{\text{argmin}} \|\mathbf{c}(\mathbf{z}) - \mathbf{y}\|;$
$\mathbf{p}_0 = I;$
for $k = 0, 1, \ldots,$
do
$\quad \mathbf{x}_{k+1} = \underset{\mathbf{x}\in X}{\text{argmin}} \|\mathbf{c}(\mathbf{p}_k(\mathbf{x})) - \mathbf{y}\|;$
$\quad$ compute updated $\mathbf{p}_{k+1}$ by means of $\{\mathbf{x}_j\}_{j=0}^{k+1};$
enddo;

Figure 3.
The SM algorithm associated with the new approach

3. Two shape optimisation experiments

3.1 Problem EPE1, a magnetic circuit

3.1.1 Introduction and analysis. EPE1 is a two-dimensional magnetostatic problem for a C-shaped circuit. It was introduced in Choi *et al.* (2001) to check the performance of the SM technique. Here we will study it in more detail and also use it to compare SM with other minimisation schemes. The success of the SM technique depends on the similarity between the models considered and their relative computational cost. In this problem it is possible to improve the accuracy of the coarse model without significantly increasing its execution time. This will result in a more efficient SM based algorithm.

A permanent magnet with residual flux density $\mathbf{B}_r = 1.0\,\mathrm{T}$ is placed between two ferromagnetic cores as shown in Figure 5. The lengths d and g are taken as 10 and 1 mm, respectively. In the first experiment with EPE1 the core is assumed to be a linear material with relative magnetic permeability equal to 5,000. The magnet is always considered linear with unit relative magnetic permeability. The design specifications are the flux densities at the air gap centre and in the core, $\mathbf{B}_g$ and $\mathbf{B}_c$, respectively, and the magnet permeance coefficient $\mathbf{P}_m$ (equivalently the magnet working point). The particular design objective is $\mathbf{y} = [\mathbf{B}_g, \mathbf{B}_c, \mathbf{P}_m] = [0.5, 1.0, 14]$. The design parameters $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3]$ are the dimensions of the magnet, $\mathbf{x}_1$ and $\mathbf{x}_2$, and the main width of the core $\mathbf{x}_3$. The control parameter spaces X and Z coincide and they are equal to the positive octant of $\mathbb{R}^3$.

The fine model is a finite element resolution of a vector magnetic potential formulation of the magnetostatic equations (Sylvester and Ferrari, 1996). The basis

$$\mathbf{x}_0 = \mathbf{z}^* = \underset{\mathbf{z}\in Z}{\operatorname{argmin}} \|\mathbf{c}(\mathbf{z}) - \mathbf{y}\|$$

$$\mathbf{B}_0 = I$$

while $\|\mathbf{p}(\mathbf{x}_k) - \mathbf{z}^*\| > \text{tolerance}$

do

$$\mathbf{h}_k = -\mathbf{B}_k^{-1}(\mathbf{p}(\mathbf{x}_k) - \mathbf{z}^*)$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{h}_k$$

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \frac{(\mathbf{p}(\mathbf{x}_{k+1}) - \mathbf{z}^*)\mathbf{h}_k^T}{\mathbf{h}_k^T \mathbf{h}_k}$$

enddo;

Figure 4.
The aggressive SM algorithm i.e. original SM with a Broyden-type approximation for $\mathbf{p}_k$. It is assumed that $\mathbf{z}^* \in \mathbf{p}(X)$. The $\mathbf{B}_k^{-1}$ should be understood as the general pseudo-inverse. Some additional simple stopping criteria should be added for a more robust performance

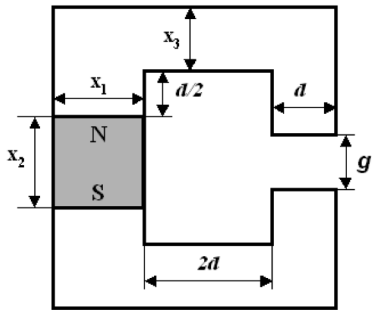

Figure 5.
EPE1: a two-dimensional magnetostatics problem

functions are Lagrangian interpolants of second-order and adaptive mesh refinement is applied, leading to discretisations of around 40,000 degrees of freedom. The software package FEMLAB (FemLab, 2004) is used for this purpose. This fine model takes into account phenomena like the fringing effect at the air gap and the flux leakage through the air central window.

The first coarse model is the same as used in Choi *et al.* (2001). The flux density is assumed to be confined to the magnet, the core and the air gap and also to be constant in each of these three regions. The magnetic permeability in the core is taken as infinity. No fringing effect around the air gap or flux leakage through the window is considered. Just by applying the divergence and Stokes' theorems to the integral form of the magnetostatic equations, we obtain the coarse model

$$\mathbf{c}(\mathbf{z}) = \left[\frac{\mathbf{B}_r \mathbf{z}_1 \mathbf{z}_2}{g\mathbf{z}_1 + d\mathbf{z}_2}, \quad \frac{d\mathbf{B}_r \mathbf{z}_1 \mathbf{z}_2}{g\mathbf{z}_1\mathbf{z}_3 + d\mathbf{z}_2\mathbf{z}_3}, \quad \frac{d\mathbf{z}_2}{g\mathbf{z}_1}\right], \tag{6}$$

where the first component approximates the flux density at the air gap, the second one the flux density in the core and the third one the magnet permeance coefficient. Computing the coarse optimum $\mathbf{z}^*$ is straightforward. It should be noted that the coarse model minimisation time can be neglected with respect to that for the fine model. The resulting value for $\mathbf{z}^*$ is shown in Table I.

3.1.2 Results. Two versions of the SM algorithm are compared with three standard optimisation schemes. Because the number of design parameters equals the number of specifications, we may expect that one point in X satisfies $\mathbf{f}(\mathbf{x}^*) = \mathbf{y}$ and, therefore, that the condition for a perfect mapping holds. Thus, the stopping criterion adopted,

$$\frac{\|\mathbf{p}(\mathbf{x}_k) - \mathbf{z}^*\|_2}{\|\mathbf{z}^*\|_2} \leq \tau, \tag{7}$$

(Choi *et al.*, 2001), with $\tau = 0.001$, makes sense in this problem for finding $\mathbf{x}^*$, even for algorithms that are not based on SM. The advantage of equation (7) over, for example, $\|\mathbf{f}(\mathbf{x}_k) - \mathbf{y}\|_2/\|\mathbf{y}\|_2$ is that in the former choice, the three vector components are well scaled (the optimal dimensions are comparable in size).

	# Evaluations	Final design (mm)
$\mathbf{z}^*$		[5.3571, 7.5000, 5.0000]
SM I	4	[7.9897, 7.5821, 6.5396]
SM II	6	[7.9797, 7.5808, 6.5381]
qN	6	[7.9891, 7.5821, 6.5394]
NMS	62	[8.0000, 7.5872, 6.5372]
DIRECT	186	[7.9806, 7.5823, 6.5432]

Table I.
Efficiency comparison between different optimisation methods applied to EPE1

Notes: SM I: the ASM algorithm; SM II: the simplified ASM algorithm with $\mathbf{B}_k = I$; qN: a quasi-Newton scheme; NMS: Nelder-Mead simplex and DIRECT: a direct search method. The first column is the number of fine model evaluations (approximately proportional to the total computing time) needed to solve the EPE1 problem for a tolerance $\tau = 0.001$. The second column represents the resulting vector of design parameters. The coarse optimum $\mathbf{z}^*$ is also included (first row)

The optimisation is solved by means of five different algorithms. ASM is implemented together with a variant in which $\mathbf{B}_k$ is taken as the identity in every iteration. A quasi-Newton method was also tried for solving $\mathbf{f}(\mathbf{x}^*) = \mathbf{y}$. The Jacobian was approximated by Broyden's method and the initial estimates for the solution and for the Jacobian were $\mathbf{z}^*$ and $\delta\mathbf{c}/\delta\mathbf{z}(\mathbf{z}^*)$, respectively. As this method uses coarse model information as well, it cannot be considered based on principles really different from SM. Since derivative information for the fine model is not easily obtained, two direct optimisation methods are used: the Nelder-Mead simplex method (NMS) (Lagarias *et al.*, 1998)[2] and the global search algorithm DIRECT (Jones *et al.*, 1993)[3]. The first of these takes the coarse optimum as the initial guess. The second one needs no starting value, but Z or a subset should be specified[4]. Table I shows the number of fine model evaluations for each method until the stopping criterion is satisfied as well as the final value of the three design parameters. It should be emphasised that the number of evaluations is approximately proportional to the total computing time. ASM turns out to perform best, and the quasi-Newton scheme seems to be still efficient. Nelder-Mead shows a slow convergence. DIRECT's global optimisation nature justifies its poor results.

In the second experiment with EPE1, the core is assumed to be made of steel and, hence, shows a nonlinear **B**-**H** characteristic. The *fine* model is refined taking into account this fact. We want to show how important the selection of the coarse model is for the SM algorithm. Therefore, we improve the previous coarse model in two different ways. First, in the part of the core where its width is $\mathbf{x}_3$, the magnetic permeability is now finite and extracted from the **B**-**H** curve (the treatment of the rest of the core seems to be of no relevance for the results). Second, the fringing effect is taken into account in the coarse model by increasing the width of the air gap d to σd, with $\sigma > 1$ a correction factor. The experiments show that taking into account the fringing effect is more

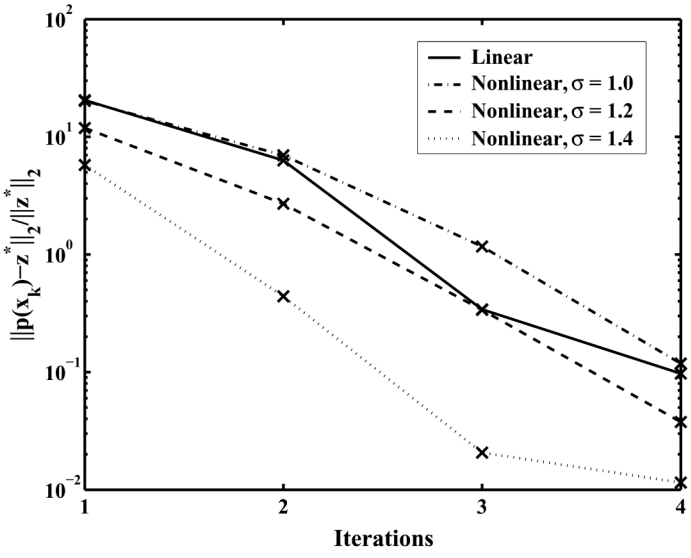

Figure 6.
Problem EPE1 solved for a nonlinear fine model. The convergence history of the cost function is shown when different coarse models are used

important than adding nonlinearities. In Figure 6, the convergence history is shown for ASM applied to the nonlinear fine model with four different coarse models. The first one is the same linear model as in equation (6), the second takes the core nonlinearities into account and the other two incorporate the fringing effect at the air gap by increasing its cross section.

We stress that all these coarse models have a negligible computational cost compared with that for the fine model. Hence, the number of SM iterations within ASM is still a good measure for the final computing time. The proper selection of the coarse model is an important key for the SM technique since convergence can be greatly improved by an adequate choice.

3.2 Problem EPE2, the coreless actuator

3.2.1 Introduction and analysis. EPE2 is a three-dimensional magnetostatic problem for an axisymmetric coreless actuator. It was proposed in Saldanha *et al.* (1992) as a test for nonlinear optimisation applied to magnetic actuators design. Here we study it with a double aim. Apart from seeing the effect of SM, compared with other efficient minimisation methods, we also want to find a proper strategy for the construction of a coarse model. In view of our experience with the EPE1 experiment, the latter is probably a delicate part. We will apply a whole hierarchy of coarse models and this will give us some insight into the question how coarse a model should be selected in a problem like EPE2.

The actuator is shown in Figure 7. It consists of a moving cylindrical magnet with length $l_m = 36\,\text{mm}$ and radius $r_m = 14.7\,\text{mm}$ and two fixed toroidal coils with inner radius $r_c = 15\,\text{mm}$ and the geometry specified by the design parameters x_1 (internal thickness), x_2 (half distance), x_3 (length) and x_4 (external thickness). We combine these four parameters in the vector $\mathbf{x}$. The design in Saldanha *et al.* (1992) is bounded by

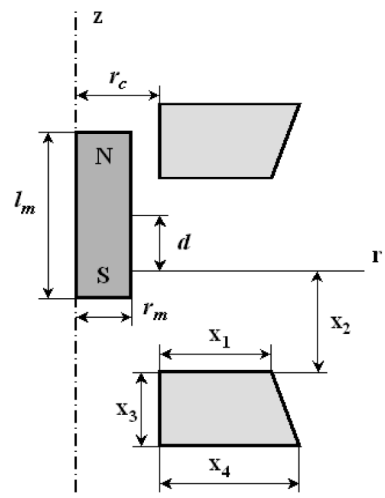

Figure 7.
EPE2: geometry of the coreless actuator

$$\begin{cases} 2.5\,\text{mm} \le x_1 \le 20\,\text{mm} \\ 5.0\,\text{mm} \le x_2 \le 25\,\text{mm} \\ 2.5\,\text{mm} \le x_3 \le 50\,\text{mm} \\ 2.5\,\text{mm} \le x_4 \le 20\,\text{mm} \end{cases} \tag{10}$$

The sets X and Z both coincide with this region.

The permanent magnet in Saldanha *et al.* (1992) has a constant residual flux density $\mathbf{B}_r = 0.35\,\text{T}$ and is assumed to be linear with a relative magnetic permeability equal to 1. The current density in the coils is taken constant, with a value $\mathbf{J}_c = 2.68\,\text{A/mm}^2$. The magnet can be moved from the centre, along its axis over some distance d.

The specification $\mathbf{y}$ is the force response exerted on the rod at different locations d, which should be constant and equal to 5 N. By *force response* we mean the force (statically) computed when the magnet is displaced over a distance d along the z-axis. For this purpose the force is determined at 20 regularly distributed displacements d_i between 0 and 15 mm from the centre. Both the specification and the computed force are vectors of length 20.

Again, finite elements are employed in the fine model. The axisymmetric vector potential formulation of the magnetostatic equations is solved using second-order Lagrangian interpolants as basis functions. The mesh is adaptively refined and the number of degrees of freedom is around 7,000-18,000. The force is computed via Lorentz's formula. This is done by post-processing after the finite element computation.

3.2.2 A hierarchy of coarse models. Although we know that the coarse model should be an easy-to-calculate approximation of the fine one, it is difficult to establish beforehand how (in)accurate it may be, still providing an acceleration of the optimisation process. In order to investigate this question, here we build a coarse model that can be tuned as (im)precise as the user desires by means of two parameters.

The first parameter, N_m, refers to the treatment to the magnet. In EPE2 the effect of the magnetization is equivalent to a surface current density $\mathbf{J}_m = \mathbf{B}_r/\mu_0$. In our coarse model this current density is approximated by N_m current loops. They are equally distributed over the vertical surface of the magnet, each with radius r_m and carrying a current $\mathbf{J}_m l_m/N_m$. The magnetic flux density is computed by the Biot-Savart law.

The second parameter, N_c, relates to the representation of the coil. Since the coil volume-current density is constant in the coil and because of the axial symmetry of the problem, the force computation is reduced to the integration of the radial component of the flux-density caused by the magnet over each coil cross-section. This cross section is divided into $(N_c - 1) \times (N_c - 1)$ pieces. The regular partition is made by $N_c - 1$ layers of $N_c - 1$ cells, as shown in Figure 8 for $N_c = 5$. The coarse model considers the flux density to be piecewise constant over this partitioning. The value for the radial flux density in every cell is computed at its baricentre. The integral over the cross-section is approximated by the sum of the calculated values, weighted by their cell area. This is equivalent to substituting the coil by $(N_c - 1)^2$ loops situated in the baricentre of each region and with a current equal to the one in that area.

Thus, a coarse model is then identified by the pair of integer parameters (N_m, N_c). As long as the values for N_m and N_c are increased, the coarse model will be more

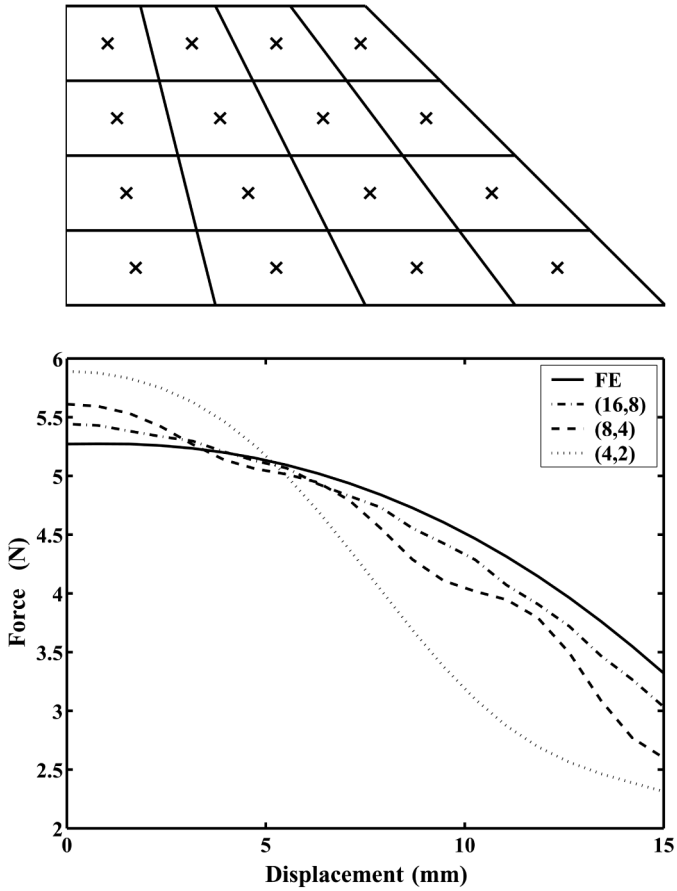

Figure 8.
A hierarchy of models for EPE2. The figure at the top shows the cells ($N_c = 5$) in the coil for which the flux density is considered constant, with its value computed in the baricentre (indicated by a cross). These points also indicate the position of the loops that approximate the coil. The figure at the bottom shows the force response for different coarse models, compared with the fine model force response $\mathbf{x}_1 = \mathbf{x}_4 = 16\,\text{mm}$, $\mathbf{x}_2 = 18.50\,\text{mm}$ and $\mathbf{x}_3 = 15\,\text{mm}$.

accurate and therefore, computationally more expensive. This behaviour is also reflected in Figure 8 where $\mathbf{x}_1 = \mathbf{x}_4 = 16\,\text{mm}$, $\mathbf{x}_2 = 18.50\,\text{mm}$ and $\mathbf{x}_3 = 15\,\text{mm}$, and the coarse models (N_m, N_c)=(4,2), (8,4) and (16,8) are used.

At first sight the response associated with model (4,2) seems very different from the one obtained with the finite element simulation. But apparently this does not influence its use as a coarse model. As will be seen in the Section 3.2.3, this very coarse model model eventually performs more efficiently than (8,4). The SM function compensates the strong model misalignment and guarantees the final accuracy.

3.2.3 Results. In this section, we compare the efficiencies for several optimisation techniques and different SM strategies. The SM technique performs best. Surprisingly, extremely inaccurate coarse models lead to very efficient optimisation schemes.

Unlike in EPE1, we cannot expect now perfect mapping. Therefore, the criterion equation (7) is not adequate for comparing SM with some other schemes. As in Saldanha *et al.* (1992), we take $\|\mathbf{f}(\mathbf{x}) - \mathbf{y}\|_2 / \|\mathbf{y}\|_2$ for the cost function. In the experiments the solution process is stopped when this measure is below 0.08.

It is interesting to notice that indeed we find solutions that satisfy the design criterion with sufficient accuracy. However, there is a whole region in the parameter space X where the fine model satisfies the specifications. We see in Table II, how the final design parameters obtained with the techniques considered are not unique. In fact it appears that there exists a whole manifold of possible solutions in X, so additional design constraints can be imposed.

The techniques analysed in Saldanha *et al.* (1992) are the penalty method (PM) and the method of the moving asymptotes (MMA). PM reduces the cost function to 0.0738 in 14 fine model evaluations and MMA to 0.0825 in only 9. Both algorithms need an initial guess. In the paper by Saldanha *et al.* (1992) two of them are considered. The given results start from the point $\mathbf{x}_1 = \mathbf{x}_4 = 16\,\text{mm}$, $\mathbf{x}_2 = 18.50\,\text{mm}$ and $\mathbf{x}_3 = 15\,\text{mm}$ and for both PM and MMA are better than those corresponding to the other starting point.

The NMS is modified (by penalising the cost function) so as to deal with the constraints. The initial guesses are again the same as in Saldanha *et al.* (1992). Now the best results are obtained when the starting point is $\mathbf{x}_1 = \mathbf{x}_3 = \mathbf{x}_4 = 18\,\text{mm}$ and $\mathbf{x}_2 = 10\,\text{mm}$. The cost function decreases to 0.0702 in 12 fine model evaluations. The performance of DIRECT is much better than in EPE1. It appears that the existence of multiple solutions makes the global search easier. The cost function drops to 0.0676 in 11 fine model evaluations. A method based on sequential quadratic programming (SQP) (Nocedal and Wright, 1999)[5] is also tested. It yields to 0.0468 in 7 fine model evaluations. The initial guess is the same as for the NMS.

Applying the SM technique entails solving two types of optimisation subproblems. The first one is the computation of the coarse optimum $\mathbf{z}^*$. Since the coarse model is very cheap to evaluate, we can afford to use global minimisation. We choose DIRECT for this purpose. The second subproblem concerns the SM function $\mathbf{p}$. Since usually this function is close to the identity in great part of the domain (see Figure 1), the computation of $\mathbf{p}(\mathbf{x})$ brings no significant problems. The NMS is used there and the coarse optimum $\mathbf{z}^*$ is always taken as the initial guess.

	# Evaluations	Cost function	Final design (mm)
PM	14	0.0738	[17.20, 24.10, 23.90, 15.87]
MMA	9	0.0825	[16.77, 23.50, 24.27, 16.48]
NMS	12	0.0702	[16.13, 20.53, 16.46, 16.51]
DIRECT	11	0.0676	[17.08, 21.67, 26.25, 11.25]
SQP	7	0.0468	[10.25, 17.50, 34.00, 10.25]
SM-(8, 4)	7.9	0.0501	[11.25, 21.75, 24.49, 17.08]
SM-(4, 2)	2.5	0.0412	[14.33, 20.02, 27.16, 10.75]
SM-(6, 2)	1.5	0.0391	[17.73, 21.67, 20.97, 11.25]

Notes: PM: penalty method; MMA: method of the moving asymptotes; DIRECT: direct search method; NMS: Nelder-Mead simplex; SQP: sequential quadratic programming and SM-(N_f, N_c): space-mapping with coarse model (N_f, N_c). The first column shows the total amount of computational work expressed in the equivalent number of fine model evaluations (approximately proportional to the total computing time). The second column shows the minimum cost function obtained. The design tolerance is 0.08, except for PM and MMA for which the comparable results were taken from Saldanha *et al.* (1992). The third column represents the final design parameters

Table II. Efficiency comparison between different optimisation methods applied to EPE2

SM iteration	**f** eval.	**c** eval.	Cost f.
SM-(8,4)			
1	1	135	0.0501
2	2	146	0.0375
3	3	156	0.0251
SM-(4,2)			
1	1	123	0.0863
2	2	142	0.0421
3	3	150	0.0228

Table III.
Three iterations of the SM technique using the coarse models (N_m, N_c)=(8, 4) and (4, 2)

Note: For every SM iteration, the number of cumulative fine and coarse model evaluations as well as the cost function is shown

We first tried the coarse models $(N_m, N_c) = (8, 4)$ and (4,2). We checked that just with their respective coarse optima $\mathbf{z}^*$, the cost function is 0.0501 for the (8,4) and 0.0863 for the (4,2). The latter already yields to 0.0421 in the next iteration. The convergence details of these schemes (the results for the first three iterations) are shown in Table III. In order to estimate the total computing time, the coarse model evaluations have now to be taken into account. This is done in Table II where all the methods applied can be compared (a fine model evaluation is approximately equivalent in computational time to 20, 200 and 300 of the coarse models (8,4), (6,2) and (4,2), respectively). Testing several coarse models in the hierarchy shows that the inaccurate (6,2) performs best in optimising EPE2: only two iterations (about 2.5 fine model evaluations) are needed to reach a cost function value of 0.0259, a value clearly acceptable in practise.

4. Conclusions

The SM optimisation technique has been presented. It accelerates the solution of an accurate but expensive minimisation problem with the aid of a simpler and faster one. The SM technique was employed before in optimisation in electromagnetics. Here, this promising performance has been corroborated by two experiments that emphasise the importance of proper coarse models. The first experiment (EPE1) shows how improving a coarse model can lead to a gain in performance. The second experiment (EPE2) uses a hierarchy of coarse models and it shows that even a very coarse approximation can accelerate the optimisation process considerably. In both experiments SM saved a significant amount of computational time when compared with other alternative optimisation techniques.

Notes

1. By this we mean that $\mathbf{p}(\mathbf{x}_1) \simeq \mathbf{p}(\mathbf{x}_2)$ implies $\mathbf{x}_1 \simeq \mathbf{x}_2$.
2. The MATLAB function *fminsearch* is used.
3. MatLab implementation by Finkel (2003) is employed.

4. In our case we take the cube centered in $\mathbf{z}^*$ and with the point $\mathbf{z}^*/2$ at a corner, as the region in which DIRECT works.
5. The MATLAB function *fmincon* from the optimisation toolbox is used.

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(D. Echeverría received the Telecommunication Engineering degree from the University of Zaragoza (Spain) in 1998. In 2002 he obtained the MSc degree in Applied Mathematics from the same university. Since April 2003 he is affiliated with the Centre for Mathematics and Computer Science (CWI) in Amsterdam (The Netherlands).

D. Lahaye received the MSc degree in applied mathematics from the Free University of Brussels (Belgium) in 1994. In 2001 he obtained the PhD degree in Computer Science from the Catholic University of Leuven (Belgium). Since September 2003 he is affiliated with the Centre for Mathematics and Computer Science (CWI) in Amsterdam (The Netherlands).

L. Encica received the Electrical Engineering degree from the University Politehnica of Bucharest (Romania) in 2002. Since August 2003 he is affiliated with the Eindhoven University of Technology (The Netherlands).

P. W. Hemker received the MSc degree in mathematics and biochemistry and the PhD degree in mathematics from the University of Amsterdam (The Netherlands) in 1971 and 1977, respectively. Since 1970 he is a researcher in the Centre for Mathematics and Computer Science (CWI) in Amsterdam. Since January 2002 he is appointed as CWI fellow, a prestigious position with complete freedom in research objectives.)